



An Investigation of the Magnetic and Structural Characteristics of Li-Ni ferrite nanoparticles

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Keywords: Crystallite Size, Spine1 Ferrites, Sol-Gel Auto-Combustion, Saturation Magnetization.

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Abstract:

Nanoparticles of Li-Ni ferrite ($\text{LiNi}_{0.5}\text{Fe}_{2.5}\text{O}_4$) were synthesized through the sol-gel auto combustion method, employing lithium nitrate, nickel nitrate, and ferric nitrate as precursors, with citric acid acting as a chelating agent. The samples were annealed at temperatures (as-burnt, from 400 to 800 °C). The impact of various calcination temperatures on the crystalline structure and magnetic characteristics of the $\text{LiNi}_{0.5}\text{Fe}_{2.5}\text{O}_4$ nanoparticles was meticulously investigated through X-ray diffraction (XRD) and a vibrating sample magnetometer (VSM).

From XRD analysis, crystallite size D was determined to be the most intense (311) peak using Scherrer's formula. An increase in crystallite size was observed with higher annealing temperatures in the range of 28.50-41.345 nm, while coercivity, with a range 47.9-154.5 Oe, showed an initial rise before decreasing as crystallite size grew. Variations in saturation magnetization (range 6.23-8.02 emu/g) and lattice constant (range 8.291-8.295 Å) displayed a similar trend, decreasing at 400 °C and 600 °C but increasing at 800 °C.

Keywords: Crystallite Size, Spine Ferrites, Sol-Gel; Auto-Combustion, Saturation Magnetization.

دراسة الخصائص المغناطيسية والتركيبية لجسيمات النانوية $Li-Ni$ الفراتية

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الخلاصة:

تم تحضير الجسيمات النانوية من فيرايت الليثيوم-نيكل ($LiNi_{0.5}Fe_{2.5}O_4$) باستخدام تقنية السول-جيل مع الاحتراق الذاتي، حيث استُخدمت نترات الليثيوم، نترات النيكل، ونترات الحديد كمواد أولية، وتم استخدام حمض الستريك كعامل مخلب. تم تلدين العينات عند درجات حرارة مختلفة (عند الحرق، من 400 إلى 800 درجة مئوية). تمت دراسة تأثير درجات حرارة التكليس بشكل منهجي على التركيب البلوري والخصائص المغناطيسية لجسيمات $LiNi_{0.5}Fe_{2.5}O_4$ النانوية باستخدام حيود الأشعة السينية (XRD) ومقياس مغناطيسية العينة الاهتزازي (VSM). من تحليل XRD ، تم تحديد حجم البلورة D لأشد قمة (311) باستخدام معادلة شيرر. لوحظت زيادة في حجم البلورات مع ارتفاع درجات حرارة التلدين ضمن نطاق 28,50-41,345 نانومتر. أظهرت القوة القسرية (Oe 154,5-47,9) زيادة أولية ثم انخفاضاً مع زيادة حجم البلورات. كما أظهرت التغيرات في المغناطيسية المشبعة (emu/g 8,02-6,23) والثابت الشبكي (A° 8,295-8,291) اتجاهًا مشابهًا، حيث انخفضت عند 400°م و 600°م، لكنها ارتفعت عند 800°م.

الكلمات المفتاحية: حجم البلورات، الفيرايت السبينيل، الاحتراق الذاتي؛ السول-جيل، المغناطيسية المشبعة.

1. Introduction:

Ferrite nanoparticles are a significant category of magnetic nanoparticles that have garnered substantial interest due to their extensive uses across many disciplines, from biomedical to industrial [1]. Ferrites have unique magnetic properties, such as the ability to retain magnetism at high temperatures, making them ideal for use in electric motors, transformers, and many other applications [2,3]. Spinel lithium ferrites are characterized by their diverse properties, which can be ascribed to their capacity to accommodate cations of various transition metals inside their lattice structure. Therefore, their structural, optical, magnetic, and electrical properties may vary. Specifically, the compound $Li_{0.5}Fe_{2.5}O_4$ exhibits exceptional features, including high saturation magnetization, low dielectric loss, and a square hysteresis loop, making it a promising magnetic material for use in hybrid nano photocatalysts [4,5]. The rapid progress of wireless communication in the twenty-first century may replace expensive magnetic materials in the fields of antennas and rechargeable lithium-ion batteries [6] because of its economical price, high Curie temperature, and elevated magnetic permeability.

Ferrite nanoparticles are metal oxides characterised by a spinel structure with the general formula AB_2O_4 , wherein A and B represent metallic cations located at distinct crystallographi

csites: tetrahedral (A site) and octahedral (B site)[7]The cations in both places are coordinated to oxygen atoms in tetrahedral and octahedral sites, respectively, as illustrated in **Figure 1**. [8]

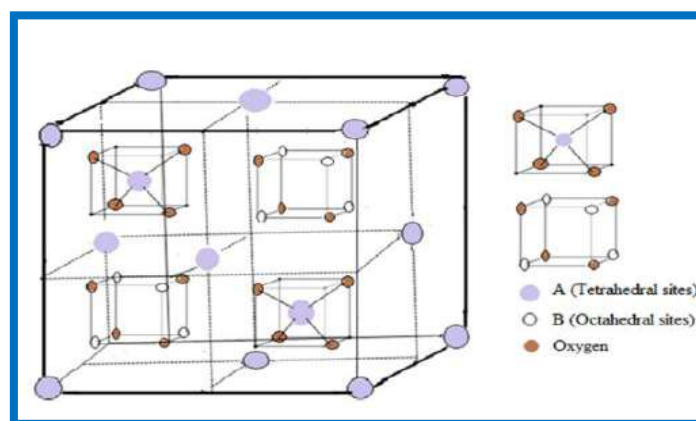


Figure1. Spinel ferrite structure showing tetrahedral and octahedral sites

For a compound to be classified as ferrite, it must contain Fe^{3+} in its chemical formula [9]. Spinel-phase nanocrystalline ferrites $\text{Li}_{0.5-0.5x}\text{M}_x\text{Fe}_{2.5-0.5x}\text{O}_4$ (where M represents Mg^{2+} , Ni^{2+} , Fe^{2+} , etc) represent one of the most prominent families of nanomaterials due to their wide range of applications, including nano ferrofluids, nanomedical devices, photocatalysts, magnetic devices, microwave devices, and gas sensors [10,11]. The ferrimagnetism of spinel ferrite is attributed to the resultant magnetic moment arising from the antiparallel alignment of magnetic moments at the A and B sites, that is, $\eta_{th} = |\eta_A - \eta_B|$. The replacement of metal cations significantly influences the microstructure and electromagnetic characteristics of lithium ferrite.

Gandomi F. et al. [12], prepared $\text{Li}_{0.5}\text{Fe}_{2.5}\text{O}_4$, $\text{Li Mg}_{0.5}\text{Fe}_2\text{O}_4$ and $\text{Li Ni}_{0.5}\text{Fe}_2\text{O}_4$ through sol-gel outo-combustion process. The doping of Ni or Mg ions reduces the remanent magnetization (Mr) and saturation magnetization (Ms) of the $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ catalyst. M.F. Warsi et al.[13] prepared praseodymium-substituted nano-crystalline Li-Ni spinel ferrites with different Pr^{3+} contents by micro-emulsion technique. The saturation magnetization (MS) of 41 emu/g and coercivity (HC) of 156.9 Oe for $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ were enhanced through the incorporation of rare earth Pr^{3+} cations.

Innovative methods such as radiofrequency inductively coupled plasma, chemical hydrothermal processes, electrochemical techniques, microwave or non-chemical processing, inert gas condensation, sol-gel auto-combustion, and mechanical milling have been developed for the synthesis of spinel ferrite nanoparticles [14]. The sol–gel auto-combustion process is a prevalent option from this list, utilized in synthesizing materials with various metastable structures, even at very low temperatures, due to its ability to produce products with excellent chemical uniformity. This technology enables the manipulation of physical attributes, including

particle size, shape, and pore structure, by altering the preparation conditions [15]. In addition to the preparation method and metal ion doping, the primary factor affecting the microstructure and performance of ferrite is the sintering temperature.

J. Song et al. [16] investigated the impact of thermal treatment on $\text{Li}_{0.35}\text{Ni}_{0.3}\text{Fe}_{2.35}\text{O}_4$ using the sol-gel auto-combustion technique. A significant increase in grain size was observed with the rise in annealing temperature. As for coercivity, the value initially increased and subsequently decreased significantly from 115 to 37 Oe with rising annealing temperature. This behavior is attributed to the grain size at 600°C approaching the transitional size between single-domain and multi-domain magnetic regions. R. Paul. Singh et al. [17], prepared magnesium ferrite (MgFe_2O_4) nanoparticles using the sol-gel technique, with the samples calcined at different temperatures. The results revealed a significant impact of calcination temperature on the structural and magnetic properties of the nanoparticles. An increase in lattice constant and crystallite size was observed with rising calcination temperature, while coercivity decreased due to the reduction of the pinning effect at the grain boundaries as the calcination temperature increased. According to reports, high sintering temperatures above 1050 °C cause lithium and oxygen to volatilize, changing the stoichiometry of lithium ferrite and so compromising its electrical and magnetic properties [18], significantly reducing the practical use of lithium ferrite.

The structural and magnetic characteristics of $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ made using the sol-gel auto-combustion process were investigated in this work. It investigates how temperature changes between 200°C and 800°C affect both its magnetic and crystalline structures.

2. Materials and Methods:

2.1 Material: $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ magnetic nano-powder was prepared using the Sol-Gel Auto-Combustion (SGAC) technique. Lithium nitrate LiNO_3 , ferric nitrate $\text{Fe}(\text{NO}_3)_3 \cdot 9\text{H}_2\text{O}$, nickel nitrate $\text{Ni}(\text{NO}_3)_2 \cdot 6\text{H}_2\text{O}$ citric acid $\text{C}_6\text{H}_8\text{O}_7$ and ammonia with mass 5.745083 g/mol of Lithium nitrate 12.11223 g/mol of Nickel nitrate, 67.3075 g/mol of ferric nitrate and 19.2124 g/mol of citric acid were used as precursors to prepare $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ magnetic nano-powders.

2.2 Synthesis $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ magnetic nano-powders: Ferric nitrate, nickel nitrate, and lithium nitrate were dissolved in deionized water at the molar ratios of 2:0.5:1, following the adjustment of the metal nitrate/citric acid ratio at room temperature. Liquid ammonia was added to the mixture to neutralize it to a pH of 7. After that, the neutralized solution was heated to 100°C on a hot plate while being continuously stirred by a magnetic device to evaporate it completely. The solution thickened as the water evaporated, eventually producing an incredibly viscous gel. The gel self-ignited when the temperature was raised to 200°C. The dried gel produced a thick, fluffy powder with a large surface area by continuing a self-sustaining

combustion reaction until it was completely burned. The as-burnt powder was subsequently calcined at various temperatures (400–800°C) for a duration of 3 hours. Experimental observations indicate that all samples exhibited combustible behavior, consistently burning entirely to produce a fine powder.

2.3 Characterization: Phase identification of the calcined powders was performed using a D/MAX-2500 X-ray diffraction (XRD) apparatus with Cu Ka radiation ($\lambda = 1.5405 \text{ \AA}$). The samples' magnetic properties were evaluated using a Vibrating Sample Magnetometer (VSM) at room temperature.

3. Results and Discussion

3.1 XRD analysis: The X-ray diffraction for $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ nano particles at various temperatures of calcination (200, 400, 600, and 800 °C) is depicted in **Figure 2**.

The results indicate that with the elevation of calcination temperature, all peaks increase concurrently, leading to narrower and sharper diffraction peaks. This signifies that the increase in particle size within the nucleus leads to a rise in both crystallization density and crystal size ratio.

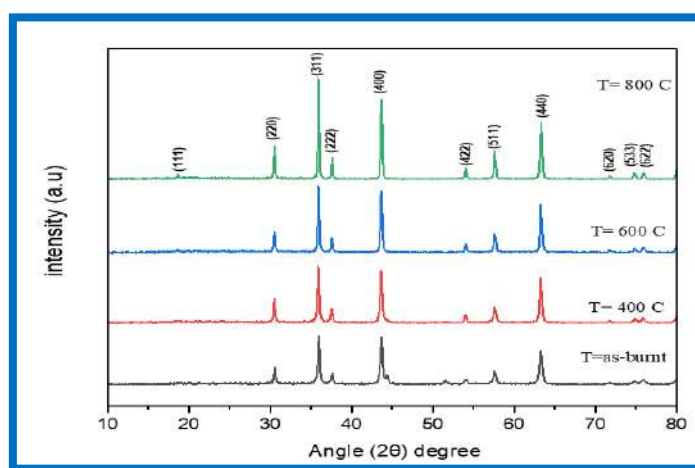


Figure 2. XRD patterns of $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ nanoparticles at various temperatures of calcination

The crystallite size (D) of Li-Ni ferrite, calcined at different temperatures, was determined from the XRD line width of the (311) peak (the greatest intensity peak) using Scherrer's equation [19].

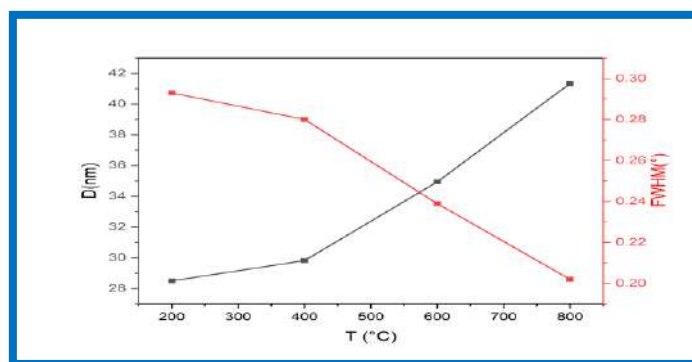
$$D = \frac{k\lambda}{\beta \cos \theta} \quad (1)$$

Where $k = 0.94$ is Scherrer's constant, $\lambda = 1.54 \text{ \AA}$ represents the X-ray wavelength, while β denotes the full width at half maximum (FWHM) of the diffraction peak, and θ is the Bragg's angle of diffraction.

Table (1): LiNi_{0.5}Fe₂O₄ nanoparticles structural characteristics for as-burnt and different calcination temperatures

Sample	T (°C)	2θ	FWHM	D(nm)	d-spacing (Å)	a (°Å)	ρ _x (g/cm ³)	LA(°Å)	LB (°Å)
LiNi _{0.5} Fe ₂ O ₄	As-burnt	35.98	0.293	28.5003	2.5007	8.294	4.934	3.5914	2.9323
	400	35.91	0.280	29.8249	2.5002	8.292	4.937	3.5907	2.9318
	600	35.94	0.239	34.9443	2.4999	8.291	4.939	3.5902	2.9313
	800	35.94	0.202	41.3458	2.5011	8.295	4.932	3.5919	2.9328

The calcination temperature significantly influenced the FWHM and crystallite size, as shown in **Table 1** and **Figure 3**. The crystallite size (D) is greatest at 800 °C due to the minimum FWHM value, resulting in enhanced crystallinity of the sample. The value rises from 28.50 nm to 41.34 nm with an increase in an annealing temperature.

**Figure 3. Variation of full width at half maximum (FWHM) and crystallite size as a function of calcination temperature.**

The lattice constant (a) exhibits an increase from 8.291 Å to 8.295 Å at calcination temperatures reaching 800 °C, as determined by the following formula [20]

$$a = d_{hkl} \sqrt{h^2 + k^2 + l^2} \quad (2)$$

Where d represents the spacing between the planes, and (h, k, l) denote the Miller indices. A small increase in the lattice constant can be elucidated using Vegard's law, based on the ionic radii of the substituent ions. The substitution of the smaller Fe²⁺ ion (0.64 Å) with the bigger Ni²⁺ ion (0.69 Å) at the octahedral sites is expected to promote an expansion of the unit cell, hence increasing the lattice constant.

X-ray density ρ_x is crucial in evaluating the electromagnetic properties of ferrites, as it is directly linked to the material's crystalline structure and atomic arrangement. These factors significantly influence its electromagnetic characteristics, such as permeability. The equation utilized to find out the X-ray density ρ_x of the samples produced is as [21]:

$$\rho_x = \frac{ZM}{Na^3} \quad (3)$$

Z denotes the number of molecules per unit cell, which is 8 for the spinel structure; M signifies the molecular weight of the corresponding composition; N represents Avogadro's number; and a indicates the lattice constant of the samples. The density (ρ_x) of Li-Ni ferrite ranged between 4.932 and 4.934 g/cm³. The X-ray density was seen to increase with temperature at 400°C and 600°C, then declining at higher temperatures due to its dependence on the lattice parameter a , as shown in Figure 4.

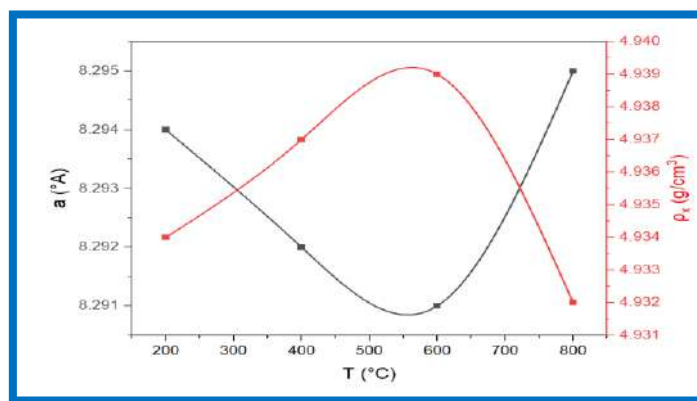


Figure 4. Variation of lattice parameter and X-ray density of the Li-Ni ferrite at various temperatures of calcination

The hopping length is the distance between magnetic ions situated in the tetrahedral (A) and octahedral (B) sites, and it may be calculated using the formulas presented below[22]:

$$L_A = 0.25a\sqrt{3} \quad (4)$$

$$L_B = 0.25a\sqrt{2} \quad (5)$$

An alteration in the hopping length on both sides was noticed with an increase in calcination temperature.

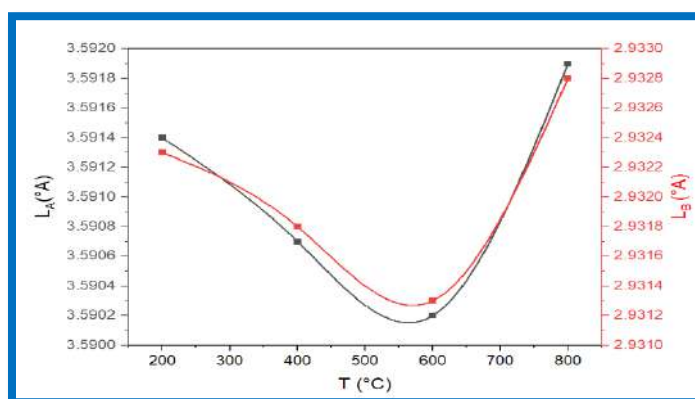


Figure 5. Influence of calcination temperature on the hopping lengths L_A and L_B for Li-Ni ferrite.

As shown in Figure 5 and Table 1, the hopping length L_A is greater than the hopping length L_B , and both vary with temperature changes. This is attributed to the spatial configuration of the tetrahedral sites, which are less constrained compared to the octahedral sites. As the temperature increases, ions such as Fe^{3+} and Ni^{2+} gain kinetic energy, facilitating shifts between

these sites. The lattice parameters are influenced by the reordering of ions, affecting the hopping distances. Furthermore, synthesis conditions, including the annealing process, contribute to the structural arrangement of ions, which in turn impacts the resulting hopping lengths and the associated magnetic properties of the ferrite material.

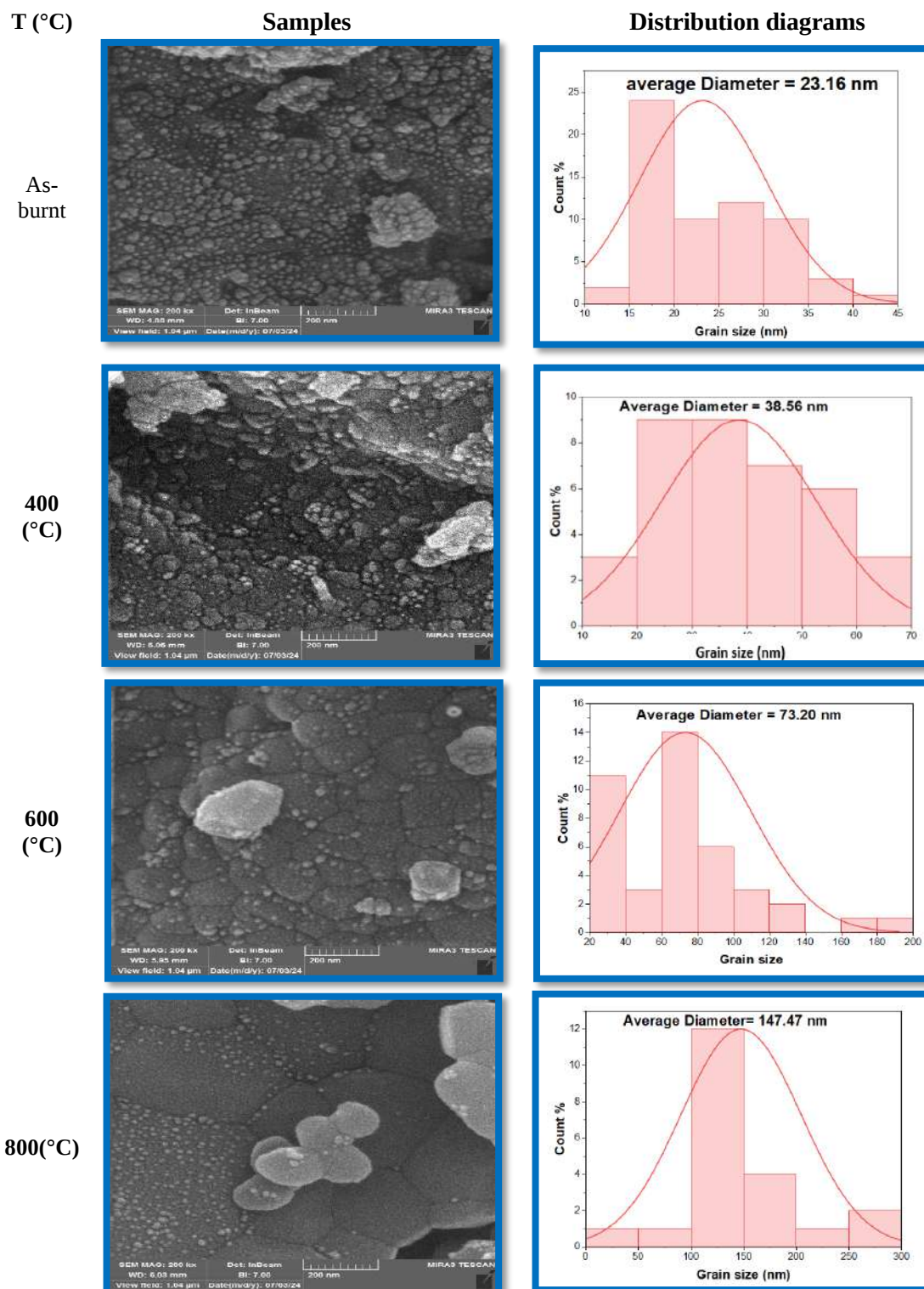


Figure 6. FE-SEM images of $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ nanoparticles at different temperatures.

3.2 FE-SEM: FE-SEM was used to examine the microstructures of the Li-Ni ferrite samples that were synthesized at different calcination temperatures (as-burnt, 400, 600, and 800 °C). The shape of ferrite samples varies from irregular hexagons to spherical aggregates, as seen in **Figure 6**. According to the particle size distribution diagrams shown in **Figure 6** at various calcination temperatures, the samples' grain sizes range from 23.16 nm to 147.47 nm, which is much larger than the values found using Scherrer's formula from XRD analysis, see **Figure 7**. This could be because of lattice strain and molecular structural disorder brought on by the various ionic radii and/or clustering of the nanoparticles. As a result, the XRD approach produces smaller sizes and has stricter criteria. Sintering frequently reduces strain and lattice defects. However, it will result in increased crystallite accumulation, which will lead to an increase in grain size. This is consistent with the pattern of $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ powder grain size at different calcination temperatures.[23]

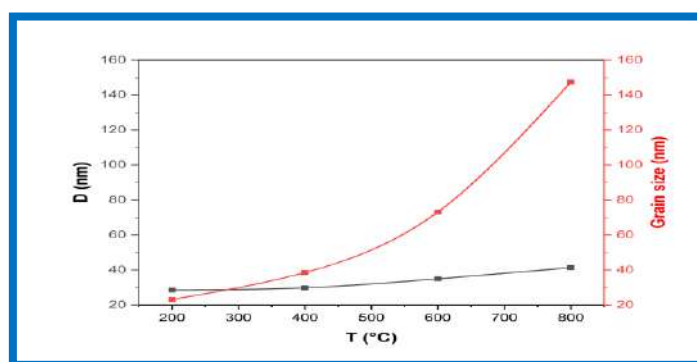


Figure 7. Variation of crystallite size determined by X-ray and grain size of the Li-Ni ferrite at various temperatures of calcination

Table 2: Magnetic properties of $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ nanoparticles for as-burnt and different calcination temperatures.

T(°C)	Ms (emu/g)	Mr (emu/g)	Hc (Oe)	Mr/Ms	k(erg/cm ³)	η _B (μB)
As-burnt	7.99	2.39	154.3	0.299	1284.2	0.303
400	6.74	2.13	154.5	0.316	1084.7	0.255
600	6.23	1.89	145.9	0.303	946.8	0.236
800	8.02	2.4	47.9	0.299	413.5	0.304

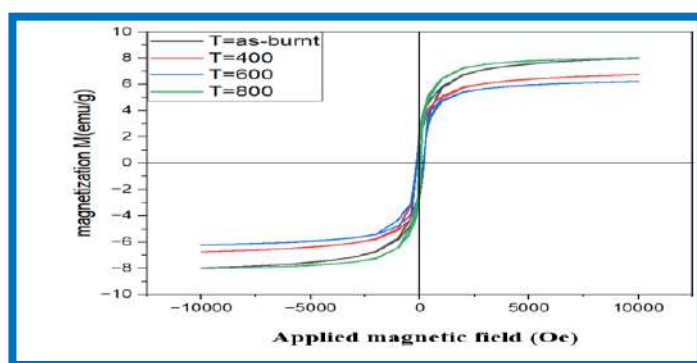


Figure 8. $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ nanoparticles hysteresis curves for as-burnt and different calcination temperatures.

3.3 VSM: The M-H loop has been constructed to examine the magnetic characteristics of as-burnt and annealed $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ nanoparticles. The maximum applied magnetic field was 10 kOe, and measurements were conducted at room temperature. The hysteresis loop provides many magnetic properties, including saturation magnetization (M_s), remanence (M_r), and coercivity (H_c), as shown in **Table 2**.

All samples exhibit soft magnetic characteristics, and the magnetic hysteresis loops get smaller and elongated with rising annealing temperatures, as illustrated in **Figure 8**. The reason is the replacement of the less magnetic Ni^{2+} ions with the magnetic Fe^{3+} ions in the octahedral sublattice of the ferrites, or the occurrence of vacancy crowding in the resulting structure. In general, Extrinsic factors (preparation process, structure, and density) and intrinsic factors (lattice parameter, lattice strain, and cation distribution) significantly affect the material's magnetic behavior.[24,25]

The vector sum of a material's magnetic moments per unit mass or volume when the material is magnetized to saturation at a particular temperature while subjected to an external magnetic field is known as saturation magnetization. Neel's model and the cation distribution can be used to explain the samples' magnetism.

This hypothesis posits that exchange interactions are responsible for the magnetic moment in spinel ferrites due to the metal cations located at the A and B sites [24]. Magnetic moment (η_B) can be expressed as

$$\eta_B = [M_w \times M_s]/5585 \quad (6)$$

Where M_w is the molecular weight of the sample, M_s is the saturation magnetization of the sample, and 5585 is the magnetic factor. A decreasing trend is observed in the saturation magnetization and magnetic moment of the nanoparticles at 400 and 600 °C. The decrease in magnetic moment is attributed to the diminished magnetic moments of the A-site (tetrahedral) and B-site (octahedral) lattices, as well as the weakened A-B interactions. The sintering temperature alters the composition of the metal contents. Ni^{2+} and Li^+ ions predominantly occupy the octahedral sites, with a minority transitioning to the tetrahedral sites as temperature rises [15]. The replacement of Fe^{3+} (5 μB) with non-magnetic lithium Li^+ (0 μB) leads to a reduction in saturation magnetization at 400 and 600°C; however, at 800°C, the arrangement of Li, Ni, and Fe ions approaches a mixed ferrite state, resulting in an increase of Fe^{3+} (5 μB) in the octahedral position and an elevation of Ni^{2+} (2 μB) in the tetrahedral position, which enhances saturation magnetization. The variations in lattice characteristics and saturation magnetization with respect to annealing temperature are almost the same. **Figure 9** illustrates

the correlation between lattice characteristics and saturation magnetization as a function of calcination temperature.

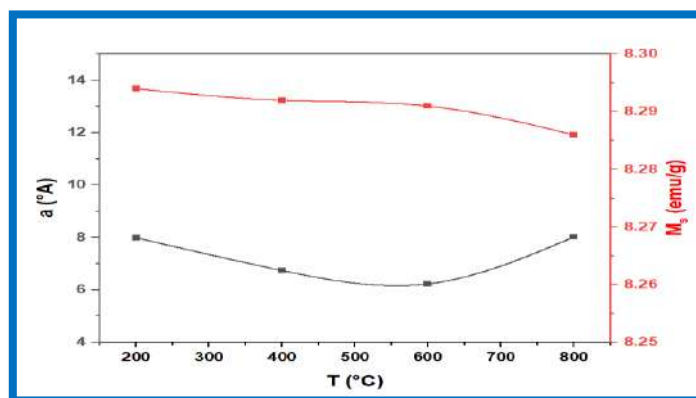


Figure 9. Correlation between lattice parameters (a) and saturation magnetization (M_s) of $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ nanoparticles as a function of calcination temperature (T).

Remanent (M_r), usually depends on (M_s), thus by decreasing the saturation magnetization in the prepared samples, remanent magnetization decreases.

Figure 10 illustrates the variation in $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ ferrites' average crystallite size (D) and coercivity (H_c) with respect to calcination temperature (T). The coercivity initially rises and subsequently declines as the crystallite size grows [26]. This variation of H_c with crystallite size can be elucidated using the principles of crystal size, single-domain or multi-domain structures, and anisotropy. The anisotropy value was determined using the following equation.:

$$k = \frac{H_c \times M_s}{0.96} \quad (7)$$

The results are presented in Table 2.

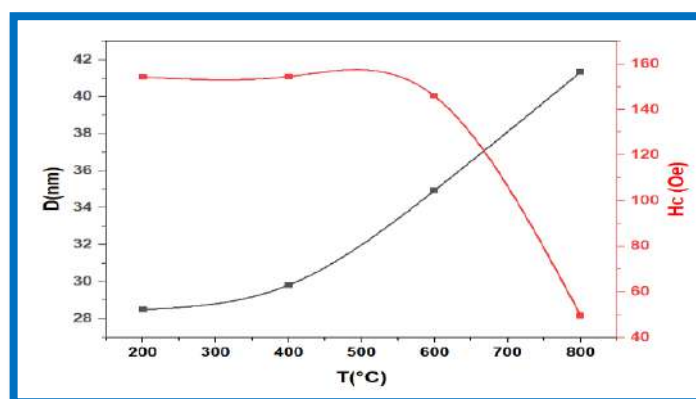


Figure 10. The fluctuation in average crystallite size (D) and coercivity (H_c) of $\text{LiNi}_{0.5}\text{Fe}_2\text{O}_4$ ferrites as a function of calcination temperature (T)

The variation in the anisotropy constant is ascribed to the shape impact of samples manufactured at varying calcination temperatures [27]. The hysteresis loops are used to calculate the remanence to saturation magnetization ratio (M_r/M_s) displayed in Table 2. The low number also suggests that the ferrites include several domains. According to the results of

this investigation, coercivity peaks at about 400 °C, which is the key temperature at which ferrites change from single-domain to multi-domain behavior.

4. Conclusions

Nano particle Li-Ni ferrites at various calcination temperatures are prepared using the sol-gel auto-combustion technique. The crystallite size D was increased with the calcination temperature up to 800 °C, which was also supported by FE-SEM studies. A maximum saturation magnetization of about 8.02 (emu/g) with a crystallite size of 41.34 nm was obtained at 800 °C, where the variation is almost the same.

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Assessing the Radioactivity of Samples Taken during a Dust Storm in the Iraqi City of Hilla

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Keywords: Specific Activity, Dust Storm, Absorbed Dose, Gamma Radiation, Nai (Tl) Detector.

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Abstract:

The research aims to investigate the distinguishing behaviors and risk indicators in samples of dust storms in Hilla city, Iraq. NaI (Tl) detector was used for the measurements, along with spectrum analysis. The findings of the measurements of elements ^{238}U , ^{232}Th , and ^{40}K 's specific activities in samples from dust storms ranged for ^{238}U from 9.2 ± 0.5 to 22.9 ± 0.8 (Bq/kg) at an average 16.92 ± 0.7 (Bq/kg), the range of ^{232}Th 's specific activity was between 2.7 ± 0.2 to 11.41 ± 0.5 at the average 6.5 ± 0.3 (Bq/kg), whereas the range for ^{40}K was 119.43 ± 2.2 to 174.92 ± 2.6 (Bq/kg) with mean 150.34 ± 2.4 (Bq/kg). Furthermore, it is shown that the radium equivalent activity and absorbed dose in dust storm samples average 37.774 Bq/kg and 34.881(nGy/h), respectively, and the external hazard index was 0.102. In contrast, the average of the effective dose of samples was 0.022 mSv/y. The findings of the present study were compared with the global average, the levels were discovered to be within the advised limit according to UNSCEAR.

Keywords: Specific Activity, Dust Storm, Absorbed Dose, Gamma Radiation, Nai (Tl) Detector.

تقييم النشاط الإشعاعي للعينات المأخوذة أثناء العاصفة الغبارية في مدينة الحلة العراقية

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الخلاصة:

يهدف البحث إلى دراسة السلوكيات المميزة ومؤشرات الخطورة في عينات العواصف الغبارية في مدينة الحلة بالعراق، وقد تم استخدام كاشف $NaI(Tl)$ للقياسات، إلى جانب تحليل الطيف. وقد تراوحت نتائج قياسات النشاط النوعي للعناصر ^{238}U و ^{232}Th و ^{40}K في عينات العواصف الغبارية لليورانيوم- ٢٣٨ من $٩,٢ \pm ٠,٥$ إلى $٢٢,٩ \pm ٠,٨$ (بيكريل/كجم) بمتوسط $١٦,٩٢ \pm ٠,٧$ (بيكريل/كجم)، وكان نطاق النشاط النوعي للثوريوم- ٢٣٢ بين $٢,٧ \pm ٠,٢$ إلى $١١,٤١ \pm ٠,٥$ بمتوسط $٦,٥ \pm ٠,٣$ (بيكريل/كجم)، في حين كان نطاق النشاط النوعي للعنصر للبوتاسيوم- ٤٠ من $١١٩,٤٣ \pm ٢,٢$ إلى $١٧٤,٩٢ \pm ٢,٦$ (بيكريل/كجم) بمتوسط $١٥٠,٣٤ \pm ٢,٤$ (بيكريل/كجم). علاوة على ذلك، تبين أن متوسط نشاط مكافئ الراديوم والجرعة الممتصة في عينات العواصف الغبارية يبلغ $٣٧,٧٧٤$ بيكريل/كجم و $٣٤,٨٨١$ (nGy/h) على التوالي، وكان مؤشر الخطر الخارجي $٠,١٠٢$. وعلى النقيض من ذلك، كان متوسط الجرعة الفعالة للعينات $٠,٠٢٢$ ميكروسيغرت/سنة. تمت مقارنة نتائج الدراسة الحالية بالمتوسط العالمي، وتبين أن المستويات تقع ضمن الحد الموصى به وفقاً للجنة العلمية التابعة للأمم المتحدة لآثار الإشعاع النووي.

الكلمات المفتاحية: الفعالية النوعية، العاصفة الغبارية، الجرعة الممتصة، إشعاع جاما، كاشف $NaI(Tl)$.

1. Introduction:

The German Association for the Building and Construction Sector has warned that small dust particles in construction sites pose a great risk to health. Sand and dust storms are an important element of Earth's natural biogeochemical cycles, but they are also influenced by human activities such as climate change, inappropriate land management, and water waste. Thus, sand and dust storms contribute to climate change and air pollution [1]. The radiation originates from a variety of sources, both natural and artificial. These include both cosmic radiation and environmental radioactivity from naturally occurring radioactive materials. Naturally, radioactivity is widespread in the Earth's environment, mainly in various geological formations and their disintegration products [2]. The major natural radioactivity sources are the nuclides of very long half-lives that have existed since the Earth's formation, and these nuclides are produced via cosmic rays [3]. The natural radionuclides of concern are mainly uranium (^{238}U), thorium (^{232}Th), or their progenies and potassium (^{40}K) [4]. Long-term exposure to radioactivity and inhalation of radioactive nuclides have serious health effects, such as

cancerous disease. Nuclides' radioactive exposure can cause lung diseases, pancreatic, hepatic, bone, and kidney cancers, and leukemia. These diseases are caused by gamma-radiation, which is capable of traveling long distances through air to affect human beings [5].

The distribution of radionuclides in the environment and their typical concentration levels in soil are of interest to many international studies to determine the levels of radioactivity in soil samples [6-10]. Numerous methods exist for uranium to enter the body, including drinking uranium-containing water, inhaling uranium-containing dust particles, or indirectly from nutrient-rich soil layers through the food chain [11]. The NaI(Tl) detectors have exceptional absorption efficiency because of the density of the crystal, whilst the detection systems based on these scintillators show high absorption efficiency for low-energy gamma ray detection due to the relatively high atomic number of iodine ($Z = 53$) [12].

There are currently no known baselines for the concentration of naturally occurring radioisotopes, and only a small number of studies on the radioactivity of soil samples in the areas under inquiry have been published. Furthermore, there have been reports of a few people of different ages who have died from cancer instances that have been documented in these areas. Therefore, the goal of this research is to examine the radioactive content of dust storm samples collected from various locations inside Hilla city during the storm. The purpose of the study was to determine whether radioactivity in the storm's dust was dangerous to people or not.

2. Area of Study

Hillah is the Babel provincial capital, which is located in central Iraq between latitudes $32^{\circ} 28' 59.99''$ N and longitudes $44^{\circ} 25' 59.99''$ E [13]. 5119 km² is that amount. Located on the Euphrates River, close to the historic city of Babel [13]. The physical arrangement of the states comprising the Babel Province is shown in Figure 1, along with the sites of sample research.

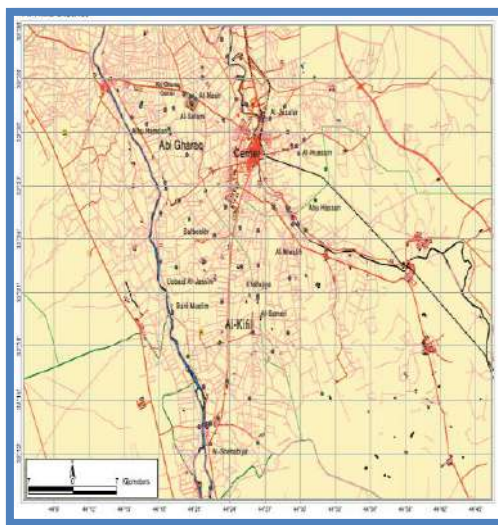


Figure 1: Study area's location on a map [13].

3. Nuclear Detection System:

Nine samples were taken from the dust storm that hit the city of Hillah in May 2022, where a deep container was placed from the beginning of the storm until its end on the roofs of the houses selected for the studied samples. After collecting the samples, they were transferred to the Advanced Nuclear Laboratory of the Physics Department at the College of Education for Pure Sciences at the University of Babylon, and were used. Natural radionuclide activity concentrations were measured using a NaI(Tl) detector comprising a "3 x 3" crystal and a Multi-Channel Analyzer (MCA) at 1024 channels. Gamma-ray spectroscopy observations were analyzed by the MAESTRO-32 software [14]. The lead and stainless steel outside section of the ORTEC cylindrical chamber is split into two pieces. These portions measured 20 cm and 5 cm in breadth, respectively. The evaluation of the whole radiation environment was made easier by the chamber's design. This detector has several advantages, including the ability to easily calibrate the power and its exceptional gamma ray capture capabilities, as **Figure 2**. An energy efficiency calibration using ^{60}Co , ^{133}Ba , ^{137}Cs , and ^{22}Na was the first step in the process, which allowed for precise energy values to be obtained.



Figure 2: Detection system

3.1 Sample Measurements: The activity concentration is calculated using the following formulae and hazard indices after determining the radiation background and calibrating the detector's energy and effectiveness.

3.2 Specific Activity (A): Calculations were made to determine the radiologic efficacy of ^{238}U using the energy of ^{214}Bi (1764 keV), the energy of ^{208}Tl (2614 keV) for ^{232}Th , and the energy of ^{40}K (2614 keV) (1460 keV). For computing radionuclide specific activity (A) in soil samples, the following equation was utilized [15]:

$$A = \frac{N.a}{I_{\gamma}.\varepsilon.m.t.} \pm \frac{\sqrt{N.a}}{I_{\gamma}.\varepsilon.m.t.} \left[\frac{\text{Bq}}{\text{kg}} \right] \quad (1)$$

,where:

N.a: refers to the area beneath each spectrum's curve after removing background radiation

I_γ: The probabilities of the transition to gamma rays.

m: The specimen's mass

t: Counting time (14400 sec)

ε: The detector's efficiency

3.3 Radium Equivalent (R_{aeq}): The levels of activity have been described by a radiological index. The formula computes the (R_{aeq}) index of (²³⁸U, ²³²Th, and ⁴⁰K) [16]:

$$(R_{aeq} \text{ Bq/kg}). = A_U + 1.43A_{Th} + 0.077A_K \quad (2)$$

where: A_U, A_{Th}, and A_K stand for the respective activity concentration of (²³⁸U, ²³²Th, and ⁴⁰K.) in (Bq/kg).

3.4 Absorbed Dose (D_r): The following equation can be used to determine the D_r in air given the abundance of terrestrial cores [17]:

$$Dr\left(\frac{nGy}{h}\right) = 0.462A_U + 0.604A_{Th} + 0.047A_K \quad (3)$$

3.5 Annual Effective Dose (AED): The predicted value was calculated using a conversion factor AED that one person of this (0.7sv/Gy) received. This was then used to determine the comparable effective dosage for a 20% outdoor occupancy and an interior occupancy of (80%) [18]:

$$AED \text{ (mSv/y)} = AD\left(\frac{nGy}{h}\right) \times 10^{-6} \times 8760 \text{ h} \times 0.2 \times 0.7 \left(\frac{sv}{Gy}\right) \quad (4)$$

3.6 External Hazard Indices (H_{ex}): For the purpose of assessing the radiation risk from gamma rays and other sources, (H_{ex}) may be computed. Depending on the level of specific radiation activity, the following two equations were obtained [17,18]:

$$H_{ex} = \frac{A_U}{370} + \frac{A_{Th}}{259} + \frac{A_K}{4810} \quad (5)$$

4. Results and Discussion:

Table 1 displays the specific activity for nine dust storm collecting samples from various zones in Babel.

Table 1: Specific activity values for ²³⁸U, ²³²Th, and ⁴⁰K

Sample No.	Specific Activity (Bq/kg)		
	²³⁸ U	²³² Th	⁴⁰ K
D1	22.91±0.8	11.41±0.5	174.92±2.6
D2	9.27±0.5	3.86±0.3	147.93±2.4
D3	16.34±0.7	10.48±0.5	169.68±2.6
D4	15.52±0.6	4.94±0.3	125.34±2.6
D5	19.67±0.7	10.31±.4	168.94±2.6
D6	16.12±0.7	6.45±0.3	123.04±2.2
D7	16.84±0.7	2.72±0.2	151.71±2.4
D8	20.61±0.8	5.91±0.3	172.15±2.6
D9	15.11±0.6	3.54±0.2	119.43±2.2
Ave± SD	16.93±0.7	6.53±0.3	150.34±2.5
Max± SD	22.91±0.8	11.41±0.5	174.92±2.6
Min± SD	9.27±0.5	2.72±0.2	117.43±2.2
UNSCEAR 2008	30	35	400

Table1. shows that the maximum and minimum values of a specific activity for ^{238}U ranged from 9.2 ± 0.5 to 22.91 ± 0.8 (Bq/kg). Sample D2 contained the lowest value, while sample D1 contained the highest value, with a mean of 16.93 ± 0.7 (Bq/kg), as **Figure 3** depicts values of activity level for ^{238}U . The current work's mean is less than 30 (Bq/kg) UNSCEAR calculated global average value is 30 (Bq/kg). Conversely, the max. and min. values of ^{232}Th a specific activity, which range from 2.72 ± 0.26 to 11.41 ± 0.5 (Bq/kg) to an average of 6.5 ± 0.3 (Bq/kg), as **Figure 4**. The current work's mean value of ^{232}Th is smaller than the mean of 35 (Bq/kg). While the maximum and minimum values of a specific activity of ^{40}K range from 119.43 2.2 to 174.92 2.7 (Bq/kg), with a mean is 150.34 2.5 (Bq/kg), as **Figure 5**.

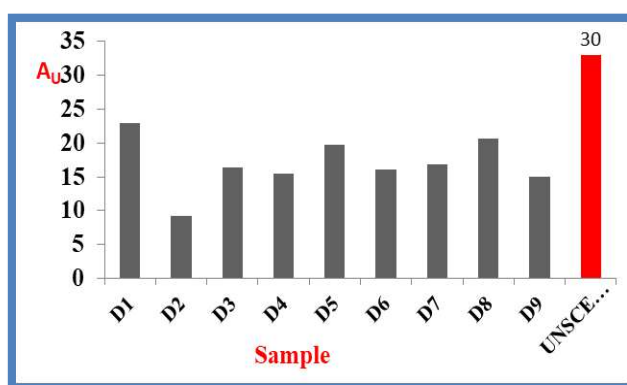


Figure 3: The specific activity of the samples' ^{238}U

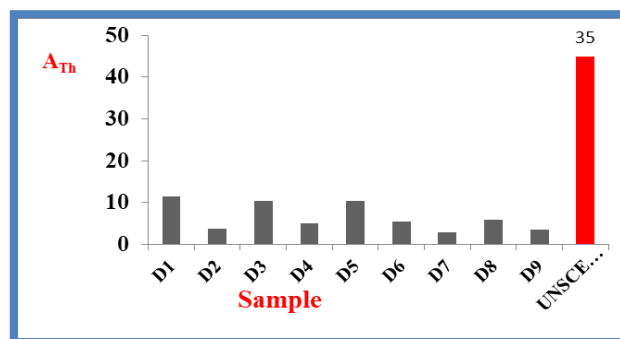


Figure 4: The specific activity of the sample' ^{232}Th

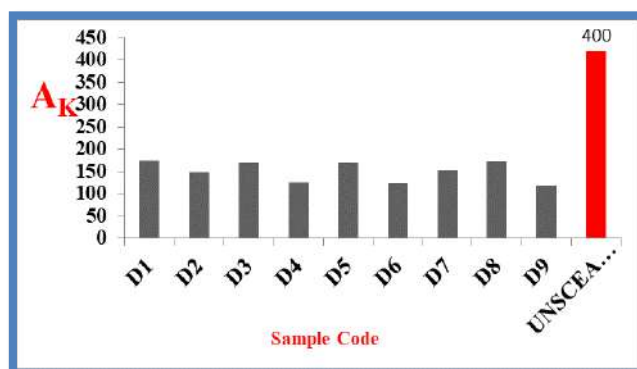


Figure 5: The specific activity of the samples' ^{40}K

Table 2 displays the Radiation Hazard Indices values, while **Figures 6** and **7** depict the Ra_{eq} and D_r , which representative level hazard indices in dust storm samples. In this study.

Table 2: Indicators of risk measured in the current work

Sample No.	Ra_{eq} Bq/kg	D_r (nGy/h)	H_{ex}	AED(mSv/y)
D1	52.685	47.789	0.142	0.029
D2	26.074	24.675	0.070	0.015
D3	44.232	40.182	0.119	0.025
D4	32.173	29.817	0.086	0.018
D5	47.368	43.080	0.127	0.026
D6	33.386	30.7911	0.090	0.018
D7	32.420	30.780	0.087	0.018
D8	42.309	39.399	0.114	0.024
D9	29.317	27.421	0.079	0.016
Ave.	37.774	34.881	0.102	0.022
Max	52.685	47.789	0.142	0.029
Min	26.074	24.675	0.070	0.015
UNSCEAR 2008	370	55	≤ 1	1

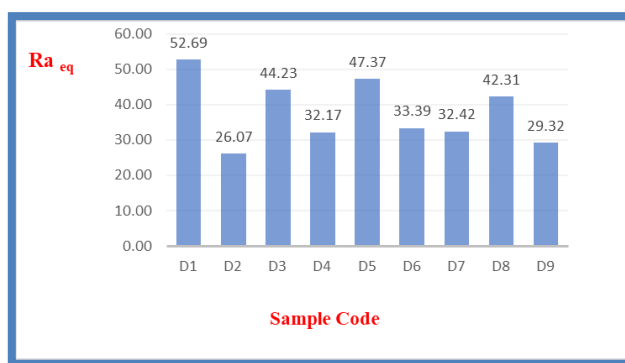


Figure 6: Radium equivalent in dust storm samples

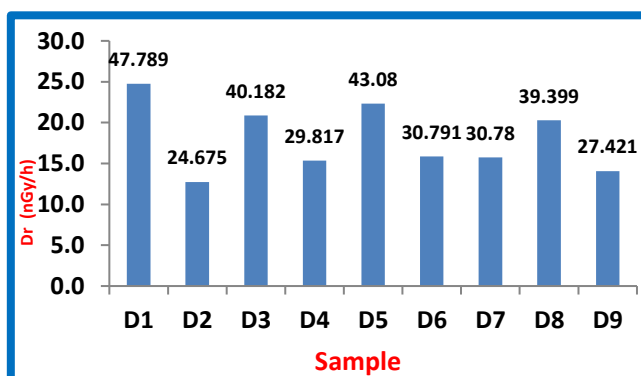


Figure 7: The absorbed dose rate in dust storm samples

It had been noticed from **Table (2)** that the Ra_{eq} values varied from 26.074 to 52.685 (Bq/kg) with an average of 37.774 (Bq/kg), as shown in **Figures (6)** and **(7)**. All dust storm samples had Ra_{eq} values that were fewer than the high value that was advised, 370 (Bq/kg), the D_r value varied between 24.675 (nGy/h) to 47.789 (nGy/h). The levels of H_{ex} differed by 0.07 - 0.142 to 0.095 - 0.204, AED values varied by 0.015 to 0.029 (mSv/y), at the average was 0.022

(mSv/y). According to the most recent data, dust storm sample readings for (R_{eq} , D_r , H_{ex} , and AED) were below the UNSCEAR and ICRP recommended value [19,20].

5. Conclusions

In the dust storm samples from the Iraqi city of Hillah, the specific activity of ^{238}U , ^{232}Th , and ^{40}K is lower than the global average values. The radiation safety publications UNSCEAR (2008) and ICRP (1993) found that the majority of the radioactive effects, including R_{eq} , D_r , H_{ex} , and AED, were below the global average. Thus, dust storms do not pose a radiation risk.

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A Novel Approach in Number Theory for Representing Large Numbers: The Arrow-Free Notation

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Abstract:

This article introduces a new notation for expressing extremely large numbers, based on the hyperoperation concept in group theory. The method employs a finite sequence of positive integers separated by specific notational symbols, allowing for concise representation through an arrow-free notation: (a_n^b) , where b represents the number of copies of a , and n denotes the arrow's number described by a general formula. This recursive definition aims to replace the Knuth up-arrow notation and Conway chained arrow notation, which require the insertion of arrows between or within numbers. The new approach simplifies these expressions, eliminating the need for such symbols and providing a straightforward and concise method for representing large numbers. The aim was to develop a more efficient method, arrow-free notation, reducing the complexity and steps necessary with previous notations.

Keywords: Number Theory, Knuth Up-Arrow Notation, Conway Chained Arrow Notation, Hyperoperation.

نهج جديد في نظرية الأرقام لتمثيل الأعداد الكبيرة: الترميز الخالي من الأسهم

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الخلاصة:

يتناول هذا المقال ترميزاً جديداً يتعلق بمفهوم العمليات الفائقة في نظرية الزمر. يتعامل مع طرق التعبير عن بعض الأعداد الكبيرة جداً. هو ببساطة تسلسل محدود من الأعداد الصحيحة الموجبة مفصولة بترميز خالي من الأسهم حيث يكون b هو عدد نسخ a ، و n هو عدد الأسهم، والذي يوصف بالصيغة العامة؛ (a_n^b) كما هو الحال مع معظم الترميزات التوافقية، فإن التعريف تكراري. في هذا المقال، ينتهي الترميز بأن يكون العدد الأيسر مرفوعاً إلى قوة عدد صحيح (عادة ما يكون ضخماً). تم تصميم هذه الطريقة لتحل محل الطريقتين المستخدمتين سابقاً لتمثيل الأعداد الكبيرة، وهما ترميز السهم الصاعد لـ Knuth وترميز السلسلة المتسلسلة لـ Conway. يبسط هذا النهج الجديد التعبير عن الأعداد الكبيرة دون الحاجة إلى كتابة الأسهم بين الأعداد أو داخلها، كما كان مطلوباً في الطرق السابقة. كان هدف هذا البحث هو تحقيق هذه الميزة بهذه الطريقة المبتكرة، التي تبسط عدة خطوات كانت مطلوبة سابقاً من الطريقتين الأخريين، مما يوفر طريقة أكثر بساطة ووضوحاً لتمثيل نفس الحالات. الطريقة بسيطة وواضحة مع خطوات قليلة، مما يجعل من السهل تعلمها وتطبيقها لمعالجة مختلف حالات الأعداد الكبيرة والكبيرة جداً.

الكلمات المفتاحية: نظرية الأعداد، ترميز السهم المتصاعد لـ Knuth ، ترميز أسهم السلسلة لـ Conway ، العمليات العددية.

1. Introduction:

The notation x^y for tetration, introduced by Hans Maurer in 1901, has evolved significantly over the years [1]. In 1947, Reuben Louis Goodstein coined the term tetration and named other hyper-operations [2]. Andrzej Grzegorzczuk further advanced this field in 1953, leading to the sequence of hyper-operations sometimes being referred to as the Grzegorzczuk hierarchy [2,3]. Rudy Rucker's 1995 publication, *Infinity and the Mind*, popularized this notation extensively [4], despite his credit sometimes being mistakenly attributed to its first use. In 1976, Donald E. Knuth developed the Up-arrow notation for hyper-operations [4], which has found significant success on the internet due to its convenience in representing tetration as " x^y ". More recently, in 1996, John Horton Conway and Richard K. Guy introduced another hyper-operation notation in their book *The Book of Numbers* [2,3]. Large numbers, which significantly exceed those typically encountered in everyday life, such as in simple counting or monetary transactions, are frequently found in fields like mathematics, cosmology, cryptography, and

statistical mechanics. In 2002, Jonathan Bowers developed Array Notation, which extends beyond hyperoperations while still encompassing them [3]. Bowers' notation includes an infix notation known as extended operator notation, equivalent in all respects to the current de facto standard notation for hyperoperations [4,5].

In mathematics, large numbers, defined as numbers exceeding one million, are typically represented either using exponents, such as (10^9 , or 10^9), or by terms like billion or thousand million, which vary across different numeration systems. The American system of numeration for denominations above one million was originally based on a French system. However, in 1948, the French system was modified to align with the German and British systems [6,7]. In the American system, each denomination above one thousand million (the American billion) is 1,000 times the preceding one (for example, one trillion equals 1,000 billion; one quadrillion equals 1,000 trillions). Conversely, in the British system, each denomination is 1,000,000 times the preceding one (for example, one trillion equals 1,000,000 billion), with the exception of the term "milliard," which is sometimes used for one thousand million. In recent years, British usage has increasingly reflected the widespread adoption of the American system [8].

Scientific notation was devised to handle the vast range of values that arise in scientific research. For example, 1.0×10^9 denotes one billion, or a 1 followed by nine zeros: 1,000,000,000. Its reciprocal, 1.0×10^{-9} , represents one billionth, or 0.000000001. Using 10^9 instead of nine zeros alleviates the reader's effort and minimizes the risk of errors associated with counting a long series of zeros to determine the magnitude of the number [9]. In addition to scientific (powers of 10) notation, the following examples illustrate the systematic nomenclature of large numbers based on the short scale. The Avogadro constant [10], for example, is the number of "elementary entities" (usually atoms or molecules) in one mole; the number of atoms in 12 grams of carbon-12 is approximately 6.022×10^{23} , or 602.2 sextillion. The lower bound on the game-tree complexity of chess, also known as the "Shannon number" (estimated at around 10^{120}), or 1 novemtrigintillion. Rayo's number, named after Agustín Rayo, is a large number that has been claimed to be the largest named number [11]. At MIT on January 26, it was first defined during a "big number duel", 2007[12]. Graham's number, which surpasses what can be represented using power towers (tetration), can nonetheless be expressed using layers of Knuth's up-arrow notation. Nevertheless, there are very specific methods used to write out such huge numbers. Gödel numbers, as well as similar huge numbers used to represent bit-strings in algorithmic information theory, are extraordinarily large, even for mathematical statements of moderate length. One of the earliest mentions of hyperoperations was by Albert Bennett in 1914, where he developed a portion of his theory on commutative

hyperoperations [11,12]. Twelve years later, Wilhelm Ackermann defined a function, denoted as (ϕ) , which resembled a sequence of hyperoperations. In a 1947 paper, the Greek mathematician Ruben Goodstein introduced a sequence of operations now known as hyperoperations [12]. He extended these operations beyond exponentiation by adding the subsequent steps, which he suggested be called tetration and pentation. As a function with three arguments, this is recognized as a variant of the original Ackermann function. Nevertheless, some pathological numbers surpass the magnitude of Gödel numbers associated with typical mathematical propositions [10,11]. In general, there are very specific methods used to write out such extremely large numbers, such as:

- Scientific Notation: A way of expressing numbers as a product of a coefficient and a power of 10 [11]. For example, 3.2×10^{15} represents 3,200,000,000,000,000.
- Knuth's Up-Arrow Notation: A technique to represent exemplifying very large integers using arrows to denote iterated exponentiation [11]. For instance, $3 \uparrow\uparrow 3$ is (3^{3^3}) , or (3^{27}) , which is (7,625,597,484,987).
- Conway's Chain Arrow Notation: An extension of up-arrow notation, useful for representing even larger numbers [12]. For example, $3 \rightarrow 3 \rightarrow 3$ represents a very large number.
- Steinhaus-Moser Notation: A way to represent very large numbers using polygons and the Ackermann function [13]. For example, 2 inside a triangle ($\Delta 2$) is equivalent to $2^{\wedge 2}$ or 4, but the notation can escalate quickly to represent vastly larger numbers.
- Hyper-E notation: A notation for expressing extremely large numbers, using "E" to denote a level of exponentiation beyond standard operations [14]. For example, $3E3$ represents (3^{27}) .
- Tree notation: Used in certain contexts like Kruskal's tree theorem [11,15], where numbers grow rapidly with tree structures and graph-related operations.

A standardized method for writing very large numbers facilitates their easy sorting in ascending order and provides a clear understanding of how much larger one number is compared to another. These notations represent functions of integer variables that grow at an exceptionally rapid rate as the integer values increase. By applying these functions recursively with large integer arguments, it is possible to construct functions that grow even more quickly. However, functions characterized by vertical asymptotes are not appropriate for defining very large numbers, despite their rapid growth. This is due to the fact that such functions necessitate arguments approaching the asymptote, which involves working with extremely small values, such as reciprocals, rather than directly addressing the construction of large numbers.

2. Materials and methods:

In this paper, we introduce Al-Ossmi's notation, a novel method for expressing extremely large numbers, named after its creator, Al-Ossmi. This notation aims to compactly represent large numbers by providing a clear structure that indicates the base, the level of iteration, and the depth of the operation. By doing so, it offers an efficient and unambiguous method for handling vast numerical values, making it a valuable tool for mathematicians and computer scientists.

Al-Ossmi's arrow-free notation is defined as; $a_n^{(b \text{ copies of } a)}$, where:

- a : The base number.
- b : The number of iterations or the height of the power tower.
- n : The level of operation or the number of arrows in Knuth's notation.

Also, the arrow-free notation developed to be extended to deal with the pentation (iterated tetration), including more complex structures, representing additional levels of nested operations where d and c are variables, such as;

$$a_n^{a_n^{(a_n^a)}}, a_n^{\left(a_n^{(a_n^{(a_n^a)})}\right)}, \text{ or } a_n^{b.c.d.e.f.g},$$

Al-Ossmi's arrow-free notation simplifies the representation of very large numbers by using a compact form that corresponds to $(a \uparrow^n b)$ as in Knuth's up-arrow notation [15], and in Conway's chained arrow notation $a \rightarrow b \rightarrow n$ [11,16].

3. Results:

The method of writing and representing extremely large numbers is essential in mathematics, as it helps simplify and understand complex calculations and allows for the precise and concise expression of enormous quantities. Using specific symbols and notations to represent these numbers facilitates their handling in various mathematical fields, enhancing the efficiency of computations and analyses. These methods provide valuable tools for mathematicians and scientists who deal with numbers that exceed traditional representation capabilities. In our research, we will focus on the two most well-known methods, Knuth's up-arrow notation and Conway's chained arrow notation.

3.1 Knuth's up-arrow notation: This notation, introduced by Donald Knuth in 1976, is a system designed for representing very large integers [15,16]. It also illustrates the representation of numbers and the execution of arithmetic operations within this base system.

Definition: For all integers a, b, n with $a \geq 0, n \geq 1, b \geq 0$, [16], the up-arrow operators can be formally defined by:

$$a \uparrow^n b = \begin{cases} a^b, & \text{if } n = 1; \\ 1, & \text{if } n > 1 \text{ and } b = 0; \\ a \uparrow^{(n-1)} b(a \uparrow^n (b-1)), & \text{otherwise} \end{cases}$$

This definition employs exponentiation as the base case and tetration as repeated exponentiation, aligning with the hyperoperation sequence but excluding the fundamental operations of succession, addition, and multiplication. The sequence begins with a unary operation (the successor function), for $(n = 0)$ and progresses through binary operations such as addition $(n = 1)$, multiplication $(n = 2)$, exponentiation $(n = 3)$, as well as tetration $(n = 4)$, and pentation $(n = 5)$, among others. This framework is often used to represent hyperoperations with arrows, for example;

- The single arrow ($\uparrow$) represents exponentiation (iterated multiplication);

$$3 \uparrow 2 = 3 \times 3 = 3^2 = 9$$

- The double arrow ($\uparrow\uparrow$) represents tetration (iterated exponentiation);

$$3 \uparrow\uparrow 4 = 3 \uparrow 3 \uparrow (3 \uparrow 3) = 3 \uparrow 3 \uparrow (3^3) = 3 \uparrow 3 \uparrow (27)$$

- The triple arrow ($\uparrow\uparrow\uparrow$) represents pentation (iterated tetration);

$$3 \uparrow\uparrow\uparrow 4 = 3 \uparrow\uparrow 3 \uparrow\uparrow (3 \uparrow\uparrow 3) = 3 \uparrow\uparrow 3 \uparrow\uparrow (3 \uparrow 3 \uparrow 3) = 3 \uparrow\uparrow 3 \uparrow\uparrow (3^{3^3})$$

The up-arrow notation general definition is as follows (for $a \geq 0, n \geq 1, b \geq 0$):

$$a \uparrow^n b = a[n+1]b$$

Here, $\uparrow$ stands for n arrows, so for example: $3 \uparrow\uparrow\uparrow 5 = 3 \uparrow^4 5$.

Square brackets are another notation for hyperoperations. Exponentiation for a natural power b is defined as iterated multiplication, which Knuth denoted by a single up-arrow:

$$a \uparrow b = a^b, \text{ for example, } 3 \uparrow 4 = 3 \times 3 \times 3 \times 3 = 3^4 = 81$$

Tetration is well-defined as iterated exponentiation, which Knuth denoted by a “double arrow”:

$$a \uparrow\uparrow b = a \uparrow (a \uparrow (\dots \uparrow a)), \text{ } b \text{ copies of } a\text{'s.}$$

Example; $3 \uparrow\uparrow 4 = 3 \uparrow 3 \uparrow (3 \uparrow 3) = 3^{(3^{3^3})} = 3^{(7,625,597,484,987)}$

Expressions are evaluated from right to left because the operators are right-associative. This results in very large numbers, but the hyperoperator sequence continues beyond pentation. Pentation, defined as iterated tetration, is represented by the “triple arrow”:

$$a \uparrow\uparrow\uparrow b = a \uparrow\uparrow (a \uparrow\uparrow (\dots \uparrow\uparrow a)), \text{ hence } b \text{ copies of } a\text{'s.}$$

Example, $3 \uparrow\uparrow\uparrow 4 = 3 \uparrow\uparrow 3 \uparrow\uparrow (3 \uparrow\uparrow 3) = 3 \uparrow\uparrow 3 \uparrow\uparrow (3 \uparrow 3 \uparrow 3)$

$$= 3 \uparrow\uparrow 3 \uparrow\uparrow (3^{3^3}) = 3 \uparrow\uparrow 3 \uparrow\uparrow (7,625,597,484,987)$$

Hexation, which is the iteration of pentation, is denoted using the “quadruple arrow” notation: $a \uparrow\uparrow\uparrow\uparrow b = a \uparrow\uparrow\uparrow (a \uparrow\uparrow\uparrow (\dots \uparrow\uparrow\uparrow a)), \text{ } b \text{ copies of } a\text{'s.}$

Example, $3 \uparrow\uparrow\uparrow 4 = 3 \uparrow\uparrow\uparrow 3 \uparrow\uparrow\uparrow (3 \uparrow\uparrow\uparrow 3) = 3 \uparrow\uparrow\uparrow 3 \uparrow\uparrow\uparrow (3 \uparrow\uparrow (3 \uparrow\uparrow 3))$,
 $= 3 \uparrow\uparrow\uparrow 3 \uparrow\uparrow\uparrow (3 \uparrow\uparrow (3 \uparrow 3 \uparrow 3))$
 $= 3 \uparrow\uparrow\uparrow 3 \uparrow\uparrow\uparrow (3 \uparrow\uparrow (7,625,597,484,987))$ and so on.

The general principle is that an n -arrow operator unfolds into a right-associative sequence of $(n - 1)$ -arrow operators. For instance, expressing $a \uparrow\uparrow b$ in traditional superscript notation results in a power tower. If b is a variable or is exceptionally large, the power tower may be denoted with ellipses and an annotation specifying the height of the tower.

$$a \uparrow\uparrow \cdots \uparrow b = a \uparrow \cdots \uparrow \underbrace{(a \uparrow \cdots \uparrow (\cdots \uparrow \cdots \uparrow a))}_{b \text{ copies of } a}$$

n

Examples:

$$3 \uparrow\uparrow\uparrow 2 = 3 \uparrow\uparrow 3 = 3 \uparrow 3 \uparrow 3 = 3^{3^3} = 7,625,597,484,987$$

$$3 \uparrow\uparrow\uparrow 3 = 3 \uparrow\uparrow (3 \uparrow\uparrow 3) = 3 \uparrow\uparrow (3 \uparrow 3 \uparrow 3), \text{ with; } 3 \uparrow 3 \uparrow 3 \text{ copies of } 3.$$

$$\text{Where; } 3 \uparrow 3 \uparrow 3 = 3^{3^3} = 7,625,597,484,987 \text{ copies of } 3.$$

$$\begin{aligned} 3 \uparrow\uparrow\uparrow\uparrow 4 &= 3 \uparrow\uparrow\uparrow 3 \uparrow\uparrow\uparrow (3 \uparrow\uparrow\uparrow 3) = 3 \uparrow\uparrow\uparrow 3 \uparrow\uparrow\uparrow (3 \uparrow\uparrow (3 \uparrow\uparrow 3)) \\ &= 3 \uparrow\uparrow\uparrow 3 \uparrow\uparrow\uparrow (3 \uparrow\uparrow (3 \uparrow 3 \uparrow 3)) \\ &= 3 \uparrow\uparrow\uparrow 3 \uparrow\uparrow\uparrow (3 \uparrow\uparrow (3^{3^3})) \\ &= 3 \uparrow\uparrow\uparrow 3 \uparrow\uparrow\uparrow (3 \uparrow\uparrow (7,625,597,484,987)) \end{aligned}$$

$$\text{where; } 3 \uparrow 3 \uparrow 3 = 3^{3^3} = 7,625,597,484,987 \text{ copies of } 3.$$

In this notation system, the expression $a \uparrow\uparrow b$ can be represented as a stack of power towers, where each level of the stack illustrates the magnitude of the level above it. If b is a variable or excessively large, this stack may be denoted using ellipses with an annotation indicating its height. Similarly, $a \uparrow\uparrow\uparrow b$ can be represented by multiple columns of these power tower stacks, where each column represents the number of power towers in the stack to its left. This up-arrow notation simplifies the representation of these diagrams while maintaining a geometric framework, such as tetration towers. Although this notation can handle very large numbers, the hyperoperator sequence extends beyond this scope. For extremely large numbers, Knuth's multiple arrows become impractical. In such cases, the n -arrow operator ($\uparrow^n$) is useful for describing sequences with a variable number of arrows, or hyperoperators. For numbers that surpass even this notation's capabilities, Conway's chained arrow notation can be employed. While a chain of three elements is comparable to other notations, a chain of four or more elements significantly enhances its capacity.

3.2 Conway chained-arrow notation: Similar to Knuth's, the chained-arrow notation has several properties that are similar to exponentiation, as well as properties that are specific to the operation and are gained from exponentiation. Conway's chained-arrow notation, invented by mathematician John Horton Conway, is a method for representing extraordinarily large numbers [12]. This notation involves a finite sequence of positive integers separated by rightward arrows, such as:

$$2 \rightarrow (3 \rightarrow 4 \rightarrow 5) \rightarrow 6 .$$

Like many combinatorial notations, Conway chained arrow notation is defined recursively. Eventually, the notation simplifies to the leftmost number being raised to a very large integer power.

Definition1: A "Conway notation" is defined as follows:

- Any positive integer can be represented as a chain of length 1.
- A chain of length n , followed by a right-arrow $\rightarrow$ and a positive integer, together form a chain of length $(n + 1)$.

Any chain represents an integer, according to the six rules below [10]. Two chains are said to be equivalent if they represent the same integer. Let a, b, n denote positive. Then:

1. An empty chain (or a chain of length 0) is equal to 1.
2. The chain a represents the number a .
3. The chain $(a \rightarrow b)$ represents the number (a^b) .
4. The chain $(a \rightarrow b \rightarrow n)$ represents the number $a \uparrow^n b$.

$$a \uparrow \uparrow b = a \rightarrow b \rightarrow n$$

Examples can become quite intricate rapidly. Here are a few simple instances:

$$2^{2^2} = 2 \uparrow \uparrow 3 = 2 \rightarrow 3 \rightarrow 2$$

$$4 \uparrow \uparrow 3 = 4 \rightarrow 3 \rightarrow 2$$

$$5 \uparrow \uparrow \uparrow 2 = 5 \rightarrow 2 \rightarrow 3$$

Arrow chains do not represent the iterative application of a binary operator. Instead, chains of other infix symbols $(a \uparrow^n b)$ can frequently be considered in fragments $(a \rightarrow b \rightarrow n)$ without a change, for example:

$$2^{3^2} = 2^9 = 2 \rightarrow (3 \rightarrow 2)$$

$$(2^3)^2 = 8^2 = (2 \rightarrow 3) \rightarrow 2$$

$$2 \uparrow \uparrow \uparrow 4 = 2 \uparrow \uparrow 2 \uparrow \uparrow (2 \uparrow \uparrow 2) = 2 \uparrow \uparrow 2 \uparrow \uparrow (2 \uparrow 2) = a \rightarrow b \rightarrow n = 2 \rightarrow 4 \rightarrow 3$$

The sixth rule in Conway's chained-arrow notation is essential. It dictates that for a sequence with four or more elements, ending in a number 2 or higher, the sequence is transformed into one of the same lengths but with a significantly larger penultimate element [12]. The last

element of the sequence is reduced, which simplifies the sequence according to Knuth's detailed procedure. This reduction process continues until the sequence is condensed to three elements, where the fourth rule completes the recursion.

4. The Arrow-Free Notation

The need for new arithmetic operations on very large numbers arises from various practical and theoretical considerations in fields such as science, technology, and mathematics. In this paper, we introduce a modern approach with an arrow-free notation that allows for the representation of numbers so large they are beyond common human experience. Al-Ossmi's arrow-free notation, named after its inventor, represents a function of integer variables that escalate at an exceptionally rapid rate as these integers increase. This notation allows for the recursive construction of increasingly faster-growing functions by applying it with large integer arguments.

Definition 2: For all non-negative integers a, b, n with $a \geq 0, n \geq 1, b \geq 0$, the Al-Ossmi's arrow-free operators can be formally defined by:

$$a_n^b = \begin{cases} a^b, & \text{if } n = 1 \\ 1, & \text{if } n > 1 \text{ and } b = 0; \\ a, & \text{if } n > 1 \text{ and } b = 1; \\ a_{(n-1)}(a_n(b-1)), & \text{otherwise} \end{cases}$$

It is important to note that Knuth did not define the "nil-arrow" operator $a \uparrow^0$, whereas Al-Ossmi's notation does include this concept. Furthermore, Al-Ossmi's notation can be extended to negative indices ($n \geq -2$) to align with the entire hyperoperation sequence, albeit with a delay in the indexing, which can be formed as:

$$a \uparrow^{(n-1)} b = a_{(n-1)}^b, \text{ for } (n \geq 0)$$

For ($n = 1$), we obtain the ordinary exponentiation, hence this definition uses exponentiation; $a_1^b = a^b$, as the base case, and tetration; a_2^b as repeated exponentiation. This approach aligns with the hyperoperation sequence but excludes the three fundamental operations of succession, addition, and multiplication. Alternatively, one may choose to define multiplication as follows:

$$\begin{aligned} a \uparrow^1 1 &= a^1 = a_1^1 = a \\ a \uparrow^1 b &= a^b = a_1^b = a^b \\ a \uparrow^0 b &= a \times b = a_0^b = a.b \end{aligned}$$

The up-arrow operation is right-associative, meaning that is, $a \uparrow b \uparrow c$, is understood to be, $a \uparrow (b \uparrow c)$, instead of, $(a \uparrow b) \uparrow c$, while it is donated by Al-Ossmi's arrow-free notation as:

$$a^{(b^{(c^d)})} = a \uparrow (b \uparrow c \uparrow d) = a \uparrow (b.c.d) = \mathbf{a}^{(b.c.d)}$$

$$a \uparrow b \uparrow c = a \uparrow (b \uparrow c) = \mathbf{a}_1^{(b \uparrow c)} = \mathbf{a}_1^{(b.c)} = \mathbf{a}^{(b.c)}$$

$$a \uparrow\uparrow b \uparrow c = a \uparrow\uparrow (b \uparrow c) = \mathbf{a}_2^{(b \uparrow c)} = \mathbf{a}_2^{(b.c)}$$

$$a \uparrow\uparrow\uparrow b \uparrow c = a \uparrow\uparrow\uparrow (b \uparrow c) = \mathbf{a}_3^{(b \uparrow c)} = \mathbf{a}_3^{(b.c)}$$

Then exponentiation becomes repeated multiplication, $a \uparrow b \uparrow c \uparrow d$ in form of $(a.b.c.d)$, which means; $a^{(b^{(c^d)})}$. The formal definition would be donated by Al-Ossmi's arrow-free notation as iterated exponentiation of a power tower of b :

$$a \uparrow b \uparrow c \uparrow d = a \uparrow (b \uparrow c \uparrow d) = \mathbf{a}_1^{(b \uparrow c \uparrow d)} = \mathbf{a}_1^{(b.c.d)} = \mathbf{a}^{(b.c.d)},$$

$$a \uparrow b \uparrow c \uparrow d \uparrow e = a \uparrow (b \uparrow c \uparrow d \uparrow e) = \mathbf{a}_1^{(b \uparrow c \uparrow d \uparrow e)} = \mathbf{a}_1^{(b.c.d.e)} = \mathbf{a}^{(b.c.d.e)},$$

$$a \uparrow\uparrow b \uparrow c \uparrow d \uparrow e = a \uparrow\uparrow (b \uparrow c \uparrow d \uparrow e) = \mathbf{a}_2^{(b \uparrow c \uparrow d \uparrow e)} = \mathbf{a}_2^{(b.c.d.e)},$$

Hyperoperations extend arithmetic operations beyond exponentiation. Specifically, exponentiation, defined as iterated multiplication for a natural power b , is denoted by a single up-arrow in Knuth's notation. In Al-Ossmi's arrow-free notation, this operation is represented when $(n = 1)$, as: (a_1^b) which is also donated by: a^b . In this new notation, expressions are evaluated from right to left due to the right-associative nature of the operators. Tetration, which is defined as iterated exponentiation, is represented in Knuth's notation using a "double arrow":

$$a \uparrow\uparrow b = \mathbf{a}_2^b = \mathbf{a}_{n-1}^{(b)} = \mathbf{a}^{(b \text{ copies of } a)},$$

For example;

$$3 \uparrow\uparrow 4 = 3 \uparrow 3 \uparrow 3 \uparrow 3 = 3^{3^{3^3}}$$

$$3 \uparrow\uparrow 4 = \mathbf{3}_2^4 = \mathbf{3}_1 \mathbf{3}_1 \mathbf{3}_1 \mathbf{3}_1 = 3^{3^{3^3}}$$

According to this definition, examples of numbers which are written by Knuth's can be rewritten out by the Al-Ossmi's arrow-free notation as following:

$$3 \uparrow\uparrow 2 = 3 \uparrow 3 = 3_2^2 = 3_1 3_1 = 3^3$$

$$3 \uparrow\uparrow 3 = 3 \uparrow 3 \uparrow 3 = 3_2^3 = 3_1 3_1 3_1 = 3^{3^3}$$

$$3 \uparrow\uparrow 4 = 3 \uparrow 3 \uparrow 3 \uparrow 3 = 3_2^4 = 3_1 3_1 3_1 3_1 = 3^{3^{3^{3^3}}}$$

$$3 \uparrow\uparrow 5 = 3 \uparrow 3 \uparrow 3 \uparrow 3 \uparrow 3 = 3_2^5 = 3_1 3_1 3_1 3_1 3_1 = 3^{3^{3^{3^{3^3}}}}$$

This process results in exceedingly large numbers, yet the hyperoperator sequence extends further. Pentation, which is defined as iterated tetration and denoted by Knuth using the "triple arrow," $a \uparrow\uparrow\uparrow b$, and by the arrow-free notation, it is simply represented as: $\mathbf{a}_3^b$. According to this definition:

structure of these numbers, including the base, height, depth, and level of operations, making it easier to understand and work with extremely large values.

4.1 Compared with Conway's chained notation: Let $(a > 1)$ is the base, $(b > 1)$ is the tower power, and $(n \geq 1)$ is the arrow's number, then Al-Ossmi's arrow-free notation can be written as:

$$a \uparrow^n b = a \rightarrow b \rightarrow n = \mathbf{a}_n^b = (\mathbf{a}_n \dots \mathbf{b} \text{ copies of } \mathbf{a}) = \mathbf{a}^{(\mathbf{b} \text{ copies of } \mathbf{a})}$$

When the number is power by n times, then the Al-Ossmi's notation presents the power tower by adding the base a , arrows number $= n$, and $b =$ number of tower of a , for example:

$$a \rightarrow a \rightarrow 1 = \mathbf{a}_1^a = \mathbf{a}^a$$

$$2 \rightarrow 2 \rightarrow 1 = \mathbf{2}_1^2 = 2^2 = 4$$

where $\mathbf{2}_1^2$ represents 2 with one level of iteration (simple exponentiation), and a height of 2.

$$2 \rightarrow (2 \rightarrow 2 \rightarrow 1) \rightarrow 2 = \mathbf{2}_2^3 = \mathbf{2}_1 \mathbf{2}_1 \mathbf{2}_1 = 2^{2^2} = 2^4 = 16$$

where $\mathbf{2}_2^3$ represents 2 with one level of iteration (simple exponentiation), and a height of 3.

$$2 \rightarrow (2 \rightarrow 2 \rightarrow 2) \rightarrow 1 = 2 \rightarrow (16) \rightarrow 2 = \mathbf{2}_2^4 = \mathbf{2}_1 \mathbf{2}_1 \mathbf{2}_1 \mathbf{2}_1 = 2^{2^{2^2}} = 65,536$$

where $\mathbf{2}_2^4$ represents 2 with one level of iteration (simple exponentiation) and a height of 4.

and from rule of (3); we can write it out as; $\mathbf{a}_2^b = \mathbf{a}^{(\mathbf{b} \text{ copies of } \mathbf{a})}$, which is applied in this example:

$$\begin{aligned} \mathbf{10}^{10^{10^{11}}} &= 10 \rightarrow 10 \rightarrow 1 \rightarrow (10 \rightarrow 11 \rightarrow 1) \rightarrow 2 = \mathbf{10}_2^{3^{(11)}} \\ &= \mathbf{10}_1 \mathbf{10}_1 \mathbf{10}_1^{(11)} = \mathbf{10}_2^{3^{(11)}} = \mathbf{10}_2^{(3.11)} \end{aligned}$$

where $\mathbf{10}_2^{3.11}$ represents 10 with 2 levels of iteration (simple exponentiation), and a height of 10 to the power of 11 and raised 3 times.

$$\begin{aligned} \mathbf{3}^{3^{(56)^2}} &= 3 \rightarrow (3 \rightarrow (56 \rightarrow 2 \rightarrow 1) \rightarrow 1) \rightarrow 1 = \mathbf{3}_1^{3^{(56)^2}} \\ &= \mathbf{3}_1 \mathbf{3}_1^{(56)^2} = \mathbf{3}^{3^{(56)^2}} = \mathbf{3}_1^{3.(3136)} = \mathbf{3}_2^{2.(3136)} \end{aligned}$$

Example1:

$$\mathbf{10}^{10^{10^{10^{10}}}} = \mathbf{10}_2^5 = \mathbf{10}_1 \mathbf{10}_1 \mathbf{10}_1 \mathbf{10}_1 \mathbf{10}_1$$

Example2:

$$\mathbf{10}^{10^{303}} = \mathbf{10}_2^{2^{(303)}} = \mathbf{10}_1 \mathbf{10}_1^{(303)}$$

Example3:

$$\mathbf{10}^{2^{303}} = \mathbf{10}_1^{2^{(303)}} = \mathbf{10}_1^{2.303} = \mathbf{10}^{2.303}$$

The difference between $\mathbf{10}_1 \mathbf{10}_1^{(303)}$, regarding $\mathbf{10}_1^{2.303}$, is in how exponentially larger the exponent of $\mathbf{a}_1^{b^{(c)}}$ compared to the exponent of $\mathbf{a}^{b^{(c)}}$. The difference between $\mathbf{10}_2^{2^{(303)}}$ and

$10_1^{2.(303)}$ lies in the magnitude of the exponent. The exponent itself, 10^{303} , is already a number with 304 digits (a 1 followed by 303 zeros). When you raise 10 to this power, you get a number with 10^{303} digits.

Al-Ossmi's free arrows notation a_n^b for expressing large numbers, a is the base number, b is the height or number of iterations in the power tower, and n is the level or number of arrows in Knuth's up-arrow notation.

Base Case ($n = 1$): Exponentiation: $a_n^b = a_1^b = a^b$

$$\text{Example: } 2 \uparrow 3 = 2_1^3 = 2 \times 2 \times 2 = 2^3 = 8$$

Two Levels ($n = 2$): Tetration (iterated exponentiation): $a_2^b = a \uparrow\uparrow b$

$$\text{Example: } 2 \uparrow\uparrow 3 = 2 \uparrow (2 \uparrow 2) = 2 \uparrow (2^2) = 2^{(2^2)}$$

From the rules of (2 & 3), we find; $2 \uparrow\uparrow 3 = 2_2^3 = 2_1(2_1 2_1) = 2^{(2^2)} = 16$

For $a = 10$, $b = 3$ (height of the tower), and, ($n = 2$), (level of operation), then form the rules of (2 and 3), we find;

$$10 \uparrow\uparrow 3 = 10_2^3 = 10_1(10_1 10_1) = 10^{(10^{10})}$$

Three Levels ($n = 3$): Pentation (iterated tetration):

$$a_3^b = a \uparrow\uparrow\uparrow b = a \uparrow\uparrow (\dots a \uparrow\uparrow a) \text{ , (with } b \text{ copies of } a), \text{ then:}$$

$$a \uparrow\uparrow\uparrow b = a_n^b = a_{(n-1)}^b \left(a_{(n-1)}^{(b \text{ copies of } a)} \right),$$

Example, according to Knuth's;

$$2 \uparrow\uparrow\uparrow 3 = 2 \uparrow\uparrow (2 \uparrow\uparrow 2) = 2 \uparrow\uparrow (2 \uparrow 2) = 2 \uparrow\uparrow (2^2) \equiv 2 \uparrow\uparrow 4$$

Then according to the Al-Ossmi's arrow-free notation's;

$$2 \uparrow\uparrow\uparrow 3 = 2_3^3,$$

$$2 \uparrow\uparrow (2^2) = 2 \uparrow\uparrow 4 = 2_2^{(2_1 2_1)} = 2_2^{(2^2)} = 2_2^{(4)} = 2_2^4$$

$$2 \uparrow\uparrow\uparrow 3 \equiv 2 \uparrow\uparrow 4 \equiv 2_3^3 \equiv 2_2^4$$

Let us prove that; $a \uparrow\uparrow b = a \uparrow (a \uparrow a) = a^{(a^a)}$, which can be written as a_2^b .

$$\text{Example; } 10_2^3 = 10 \uparrow\uparrow 3 = 10 \uparrow (10 \uparrow 10) = 10 \uparrow (10^{10}) \equiv 10^{(10^{10})}$$

$$10_2^3 = 10_1(10_1 10_1) = 10_1(10^{10}) = 10^{(10^{10})} = 10 \uparrow\uparrow 3$$

By Al-Ossmi's arrow-free notation definition, represents $a \uparrow^n b$ as; a_n^b , which matches our calculation above. Al-Ossmi's notation provides a concise and efficient way to represent very large numbers, combining the simplicity of Knuth's up-arrow notation with a clear structure for understanding the depth and height of power towers.

Table 1: A set of detailed examples written by Al-Ossmi's arrow- free notation:

<i>Al-Ossmi's notation</i>	<i>Interpretation</i>
10_1^2	$10 \uparrow 2 = 10^2 = 100$
10_2^2	$10 \uparrow\uparrow 2 = 10^{10} = 10,000,000,000$
10_2^3	$10 \uparrow\uparrow 3 = 10 \uparrow 10 \uparrow 10$
10_2^4	$10 \uparrow\uparrow 4 = 10 \uparrow 10 \uparrow 10 \uparrow 10$
10_3^3	$10 \uparrow\uparrow\uparrow 3 = 10 \uparrow\uparrow 10 \uparrow\uparrow 10$
10_2^6	$10 \uparrow\uparrow 6 = 10 \rightarrow 6 \rightarrow 2$
10_3^{11}	$10 \uparrow\uparrow\uparrow 11$, This is simply 10 raised to the power of 11.
10_1^{64}	$10^{4^3} = 10 \uparrow (4 \uparrow 3) = 10 \uparrow 64 = 10 \rightarrow (64) \rightarrow 1$
3_2^3	$3 \uparrow\uparrow 3 = (3 \uparrow 3 \uparrow 3) = (7,625,597,484,987)$
$4_3^2 = 4_2^4$	$4 \uparrow\uparrow\uparrow 2 = 4 \uparrow\uparrow 4 = (4 \uparrow 4 \uparrow 4 \uparrow 4) = 4^{4^{4^4}}$
5_3^4	$5 \uparrow\uparrow\uparrow 4 = 5 \uparrow\uparrow 5 \uparrow\uparrow (5 \uparrow\uparrow 5)$, it is a tetration of 5 repeated 4 times.
3_4^3	Graham's Number G: $3 \uparrow^4 3 = 3 \rightarrow 3 \rightarrow 4$
$3_{G_n}^3$	$G_{n+1} = 3 \uparrow^{G_n} 3$ starting from G_1 to G_{64}
$3_{G_{63}}^3$	$G_{n+1} = 3_3^{G_n}$, for 64 steps: $G_{63+1} = 3_3^{G_{63}}$
$3_{G_{64}}^{10}$	$3 \uparrow^{G_{64}} 10 = 3 \rightarrow 10 \rightarrow G_{64}$
5_{34}^4	$5 \uparrow^{34} 4 = 5 \rightarrow 4 \rightarrow 34$
$5_{34}^{(411^{300^{12}})}$	$5 \uparrow^{34} (411 \uparrow^{300^{12}}) = 5 \rightarrow (411 \rightarrow 300 \rightarrow 12) \rightarrow 34$
$10_2^{(3^{4^{2^5}})}$	$10^{10^{(3^{4^{2^5}})}} = 10 \uparrow 10 \uparrow (3 \uparrow 4 \uparrow 2 \uparrow 5)$
$10^{6.12.200.3}$	$10^{6^{12^{200^3}}} = 10 \uparrow (6 \uparrow 12 \uparrow 200 \uparrow 3)$
<i>Al-Ossmi's notation</i>	<i>Interpretation</i>
$10^{3.4.3402.5.3.4.2001}$	$8^{3^{4^{3402^{5^{3^{4^{2001}}}}}}} =$ $= 8 \uparrow (3 \uparrow (4 \uparrow (3402 \uparrow 5 \uparrow 3 \uparrow 4 \uparrow 2001)))$
$10_2^{10.10.303}$	$10 \uparrow\uparrow 10 \uparrow 10 \uparrow 303 = 10 \uparrow\uparrow (10 \uparrow 10 \uparrow 303)$
$100_1^{10^3} = 100_1^{1000}$	$100^{1000} = 100^{10^3}$
100_2^5	$100^{100^{100^{100^{100}}}}$
$100_2^{4^{(12)}} = 100_2^{4^{12}}$	$100^{100^{100^{100^{12}}}}$
$10_2^{3 \times 10.(3,000,000,003)}$	$10 \uparrow\uparrow (3 \times 10 \uparrow 3,000,000,003)$

Let's apply the new Al-Ossmi's free arrows notation in case of $a_n^{b,c}$, to describe such the number: $a^{(a^{a^c})}$. The given number is a power tower with the base a and height 3 exponents, with the topmost exponent being c . From the notation definition, the height or number of iterations in the power tower is b , and n is the level or number of arrows (exponentiation depth).

Example and interpretation: $a_2^{(b \text{ copies of } a)^{(c)}} = a_2^{(b \text{ copies of } a).c}$.

Base (a) = 10

Height (b): The height of the power tower here includes the topmost exponent and any additional exponents as iterations; (a^{a^c}):

Since the topmost exponent is $10^{(10^{10^{12}})}$, we adjust the height to reflect this deep nesting. Level (n): Since this is straightforward exponentiation (second level of up-arrow), $n = 2$.

We describe it in a notation reflecting $(3^{(12)})$ height, combining the depth and extending beyond simple iteration count. If we consider it as an iteration extending beyond the simple height, we express it as: $10^{10^{10^{12}}} = 10_2^{3^{(12)}} = 10_2^{3.12}$,

This flexibility of Al-Ossmi's notation with forms; $a_n^{b.c}$ or $a_n^{b.c.d.e.f.g.h}$ allows for a compact and structured way to represent extremely large numbers. These ($b.c.d.e$) are components to describe complex and extended depth and extend the depth further, representing additional layers of power towers or nested operations. By accurately reflecting the base, height (with nuanced depth like $b.c.d$, indicating an extended height in the power tower), and level of exponentiation, it simplifies understanding and interpreting large numbers. The use of multiple components separated by dots allows for a detailed and nuanced representation of the structure of the large number. This notation captures the nuances of the structure of these numbers, including the base, height, depth, and level of operations, making it easier to understand and work with extremely large values.

The original estimate is then when ($n > 1$), the value of b indicates to the tetration of the base, a . More precisely, the examples:

$$2_1^3 = 2 \uparrow 3 = 2^3 = 8,$$

$$\text{whereas; } 2_2^3 = 2 \uparrow\uparrow 3 = 2 \uparrow (2 \uparrow 2) = 2^{(2^2)} = 16$$

$$\text{note that; } 2_3^3 = 2 \uparrow\uparrow\uparrow 3 = 2 \uparrow\uparrow (2 \uparrow\uparrow (2 \uparrow 2 \uparrow 2)) = 2 \uparrow\uparrow (2 \uparrow\uparrow (2^{2^2}))$$

Therefore, compared with notations such as Conway's chained and Knuth's, the value of n is related to the number of arrows, while it is by Al-Ossmi's notation indicates that we deal with a tetration process, thus value of n in Al-Ossmi's does not help to determine the exact value of the number. Al-Ossmi's arrow-free notation easily helps to write out extremely large power towers, as it is listed in [Table 2](#) and [3](#).

Table 2: Systems of key Notations for Arithmetic Operators.

Arithmetic	Standard	Ackermann's	Knuth's	Conway's	Al-Ossmi's
Exponentiation	a^b	$\text{ack}(a,b,2)$	$a \uparrow b$	$a \rightarrow b \rightarrow 1$	a^b
Tetration	${}^b a$	$\text{ack}(a,b,3)$	$a \uparrow\uparrow b$	$a \rightarrow b \rightarrow 2$	a_2^b
Pentation	b^a	$\text{ack}(a,b,4)$	$a \uparrow\uparrow\uparrow b$	$a \rightarrow b \rightarrow 3$	a_3^b
Hexation	-	$\text{ack}(a,b,5)$	$a \uparrow\uparrow\uparrow\uparrow b$	$a \rightarrow b \rightarrow 4$	a_4^b
Fundamental rule	-	$\text{ack}(a,b,n)$	$a \uparrow^n b$	$a \rightarrow b \rightarrow n$	a_n^b

Where:

a, b, n are positive integers, hence:

a is the base number,

b copies of a ,

n is the arrow number.

Table 3: Al-Ossmi's free arrows notation of a set of extremely huge numbers in form of titration exponential express.

<i>Number Name</i>	<i>Exponential Notation</i>	<i>Al-Ossmi's notation</i>
Skewes number	$10^{10^{10^{34}}}$	$10_2^{3.34}$
Pentalogue	$10^{10^{10^{10^{10}}}}$	10_2^5
Millyllion	$10^{2^{1002}}$	$10_1^{2.1002}$
Gigillion	$10^{3 \times 10^{3,000,000,000+3}}$	$10_1^3 \times 10^{3,000,000,000+3}$
Ecetonplex	$10^{10^{303}}$	$10_2^{2.303}$
Heskironduplex	$10^{10^{10^{600}}}$	$10_2^{3.600}$
Googolduplexichime	$10^{10^{10^{1000}}}$	$10_2^{3.1000}$
Guppyduplexitoll	$10^{10^{10^{2000}}}$	$10_2^{3.2000}$
Googolduplexibell	$10^{10^{10^{5000}}}$	$10_2^{3.5000}$
Millinillion	10^{3003}	10_1^{3003}
Millinillinillion	$10^{3000003}$	$10_1^{3000003}$
Hepta-taxis	$10 \uparrow \uparrow \uparrow 7$	10_3^7
Hexa-taxis	$10 \uparrow \uparrow \uparrow 6$	10_3^6
Penta-taxis	$10 \uparrow \uparrow \uparrow 5$	10_3^5
Boogafive	$5 \uparrow \uparrow \uparrow 5$	5_3^5
Tetra-taxis	$5 \uparrow \uparrow \uparrow 4$	5_3^4
Gigaexpofaxul	$10 \uparrow \uparrow \uparrow (5 + 98)$	$10_3^{(5+98)}$
<i>Two</i>	$2 \uparrow 1$	$2_1^1 = 2^1 = 2$

5. Conclusions

In this paper, we introduce a novel notation for expressing extremely large numbers, named Al-Ossmi's notation after its creator. This notation aims to compactly represent large numbers by providing a clear structure that shows the base, the level of iteration, and the depth of the operation. By doing so, it offers an efficient and unambiguous method for handling vast numerical values, making it a valuable tool for mathematicians and computer scientists.

Al-Ossmi's arrow-free notation is defined as; a_n^b , $a_n^{b,c}$, or $a_n^{b,c,d,e}$, where:

- a : The base number.
- b, c, d, e etc.: The number of iterations or the height of the power tower.
- n : The level of operation or the number of arrows in Knuth's notation.

The original estimate is then this notation can be extended to include more complex structures, such as a_n^b , $a_n^{b,c}$, or $a_n^{b,c,d,e,f,g}$, to represent additional levels of nested operations, where d and c , are variables. Al-Ossmi's arrow-free notation simplifies the representation of very large numbers by using a compact form that corresponds to $(a \uparrow^n b)$ in Knuth's notation

and $a \rightarrow b \rightarrow n$ in the Conway's chained arrow notation. It combines these notations into a concise and easily readable format, reducing the complexity and length of numerical expressions. This notation is more standardized and better recognized within the mathematical community, making it effective for communicating and working with extremely large numbers. It is less cumbersome than writing multiple up-arrows or chaining arrows and is easy to write and understand once the rules are clear. To facilitate the adoption of Al-Ossmi's notation, detailed documentation and examples are provided. This includes practical applications in various fields such as physics, astronomy, and large number theory, where extremely large numbers are common. Al-Ossmi's arrow-free notation utilities in different fields, in physics or astronomy, this notation can simplify expressions and calculations involving vast quantities. In combinatorial mathematics or proofs involving large number theory, it provides clarity and precision.

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Approximate Solutions of Nonlinear Integral Equations Using the Cubic B-Spline Scaling Method

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Keywords: Fixed Point Method, Non-Linear Fredholm Integral Equation, Cubic B-Spline Wavelets, Scaling Functions, Darbo Condition.

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Abstract:

This paper examines a category of general nonlinear integral equations. These equations also include many special cases, such as functional equations and nonlinear integral equations of the Volterra type. In order to approximate the solutions to numerous physical, chemical, and biological issues, we implemented an approach that incorporates the fixed-point method and semi-vertical cubic scaling functions. We also obtain a numerical solution to the integral equation. Numerical examples illustrate the accuracy and validity of this method.

Keywords: Fixed Point Method, Non-Linear Fredholm Integral Equation, Cubic B-Spline Wavelets, Scaling Functions, Darbo Condition.

الحلول التقريبية للمعادلات التكاملية غير الخطية باستخدام طريقة قياس الشريحة B المكعبة

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الخلاصة:

تتناول هذه الورقة فئة من المعادلات التكاملية غير الخطية العامة. تتضمن هذه المعادلات أيضًا العديد من الحالات الخاصة، مثل المعادلات الوظيفية والمعادلات التكاملية غير الخطية من نوع فولتيرا. من أجل تقريب الحلول للعديد من القضايا الفيزيائية والكيميائية والبيولوجية، قمنا بتنفيذ نهج يتضمن طريقة النقطة الثابتة ودوال التحجيم التكميلي شبه العمودية، نحن نحصل أيضًا على حل عددي للمعادلة التكاملية. توضح الأمثلة العددية دقة وصلاحية هذه الطريقة.

الكلمات المفتاحية: طريقة النقطة الثابتة؛ معادلة فريدهولم التكاملية غير الخطية؛ الموجات المكعبة ذات شكل متعرج؛ وظيفة القياس، شرط داربو.

1. Introduction:

An integral equation is an equation with an unknown function, $x(s)$, under the integral sign [1-4]. The conventional form of this Equation in $x(s)$ is as follows.

$$x(t) = f(t) + \lambda \int_{g(t)}^{f(t)} k(t, s)u(s) ds, \quad (1)$$

$k(t, s)$ is a function that consists of two variables and is referred to as the kernel of the Equation, while λ is a constant parameter. The limits of integration are $\beta(t)$ and $g(t)$. The function $x(s)$ appears, defined under the integral sign, as well as the interior and exterior of the sign. The functions $f(t)$ and $k(t, s)$ have been given previously, and the limits of integration $g(t)$ and $f(t)$ can be constants, variables, or a combination of a constant and a variable. Integral equations have multiple forms, and there are two ways to distinguish the Equation, which depend on the limits of integration.

- 1- If the limits of integration are constant, the Equation is referred to as the Fredholm equation and is expressed in the following representation.

$$x(t) = f(t) + \lambda \int_a^b k(t, s)u(s) ds \quad (2)$$

- 2- When one of the limits of integration is a constant, and the other is a variable, the equation is considered a Volterra equation and is expressed in the following manner.

$$x(t) = f(t) + \lambda \int_a^x k(t, s)u(s) ds, \quad (3)$$

Additionally, two varieties of equations are contingent upon the form of the function $x(s)$, which is defined as follows:

1- The integral Equation is referred to as a Volterra or Fredholm equation of the first kind if the unknown function $x(s)$ is present exclusively within the integral sign.

2- A Volterra or Fredholm equation of the second kind is defined as an equation in which

The unknown function $u(s)$ is present both within and outside the integral sign. Suppose the Function $f(s)$ equals zero in Volterra or Fredholm equations. The integral Equation is referred to as a homogeneous equation. The Fredholm integral equation of the second kind is called nonlinear if the function $x(s)$ that appears under the integral sign is nonlinear. Additionally, it is expressed in the subsequent manner ($e^{-\cos^2(\frac{1}{2})} \cdot \sin^2(x) \dots$), etc. Accordingly, the Equation has been formulated as follows.

$$x(t) = f(t) + \lambda \int_{g(t)}^{f(t)} k(t, s) F(u(s)) ds, \quad (4)$$

We want to clarify in this introduction that Fredholm's integral equations can be derived from boundary value problems, and it is essential to remember Eric Fredholm's work on integral equations and the applied theory from the year (1866-1927). The Swedish scientist developed the theory of integral equations, and his research paper, presented in 1903 in the Acta Mathematica, played a fundamental role in establishing operator theory. Integral equations play a prominent role in applied mathematics, and non-linear integral equations have significant practical importance, as shown by numerous studies in the field of knowledge, encompassing biology, traffic theory, optimal control theory, economics, and other engineering sciences [5-8]. Numerous sources have examined functional integral equations' existence and analytical behaviour [5, 9, 10] using non-compactness measure techniques and fixed-point theories. In references [10], scholars Jalilian and Aghanjani presented numerous results related to the existence and unified universal gravity and the local gravity of solutions to the functional integral Equation.

$$x(t) = (Kx)(t) = f\left(t, x(\alpha(t)), \int_0^{\beta(t)} u(t, s, x(\gamma(s))) ds\right), \quad t \in [0, \infty] \quad (5)$$

These results were presented through the measure of non-compactness. These results were reached by the scientists Jalilian and Aghanjani, who worked to improve and expand upon the findings that emerged in other studies. Most functional integral equations are not amenable to analytical solutions; therefore, numerical methods are indispensable. Consequently, numerical methods are implemented to ascertain an Approximately calculated solution. The numerical solution of integral equations can be approached using projection, iterative, and Nystrom methods [11-12]; the references include the definitions of the collocation approaches [13, 14-17]. Galerkin methods are used to find numerical solutions for Fredholm integral equations, as outlined in the references [13, 14, 17-21] Spline functions, wavelets, product integration,

homotopy analysis approaches, homotopy perturbation, Adomian decomposition method, interpolation of polynomials methods, suboptimal trajectories, and multigrid methods are all viable alternatives. The Nystrom procedures are mentioned in the references [11-14, 21, 22]. In a few articles, the approximate numerical representation of the solution has been analyzed. Composition techniques are the foundation of Numerical approaches to solving functional integral equations [23-30], homotopy perturbation methods [25,26-32], Lagrange and Chebyshev interpolation methods [27,28,32-38]. The various studies in most numerical methods addressed by previous research transform the integral Equation into a linear or non-linear algebraic equation system. This paper presents a numerical solution for an integral equation utilizing a hybrid iterative approach that combines the fixed-point method with trapezoidal scaling functions. The method does not rely on any equation-solving system. The objective of employing this method for non-linear functional integral equations is to achieve a more precise solution with less error. We successfully attained favorable outcomes with this strategy and further elaborated on the findings presented in other investigations. The investigation is structured as follows: In the first section, the scholars Jalilian and Aghanjani provide an introduction to integral equations and a definition of equation number (5). The second section of the research provides the definitions necessary for effectively composing this scientific paper. In the third section, we examined several findings that pertain to having existed and the allure of the aforementioned integral Equation. In the fourth section, we introduced the development of cubic B-spline functions within the interval [0.1], as documented in sources [39-42]. In the fifth section, we provide an explanation of the strategy used in this research's solution method and the method by which we approach the genuine solution. Conversely, in the final section, we provided numerical examples to demonstrate the precision of this methodology and contrasted the accuracy of these numerical results with those from prior research.

2. Background Concepts

We provide some definitions and findings in this section relevant to the rest of the paper. $BC(\mathcal{R}_+)$ is a branch with limited space, but operations continue on $\mathcal{R}_+$, furnished with a conventional standard. $\|z\| = \sup\{|z(t)| : t \in \mathcal{R}_+, \}$. Let E be an infinite-dimensional Banach space containing the zero element. element θ and norm $\|\cdot\|$ Indicate the closed ball with radius r and center at x by writing $B(z, r)$. Closure and convex property of Z , a nonempty subset of E , are shown by the symbols $\bar{Z}$ and $\text{Conv } Z$, respectively. Let also m_E be all relatively compact combinations: their family and n_E denote the family of all nonempty bounded subsets of E . We employ the concept found in [4] for the non-compactness metric.

Definition 1. When a mapping $v: m_E \rightarrow \mathcal{R}_+$ meets specific requirements, it can indicate non-compactness in E .

1. a family $\ker u = \{X \in m_E : u(X) = 0\}$ is not-empty and $\ker x \subset n_E$.
2. $X \subset Y \Rightarrow v(X) \leq v(Y)$.
3. $u(Z) = u(Z)$.
4. $u(\text{Conv } x) = u(x)$.
5. $u(\lambda X + (1 - \lambda)Y) \leq \lambda u(x) + (1 - \lambda)u(Y)$ for $\lambda \in [0, 1]$.
6. write an equation of closed originating from sets (X_n) from n_E such that $X_{N+1} \subset X_n (n = 1, 2, \dots)$ and if $\lim_{n \rightarrow \infty} x(X) = 0$ consequently, the intersection emerged. $X_\infty = \bigcap_{n=1}^{\infty} X_n$ is nonempty. Given the Example of Banas and Goebel, we present a Darbo-type fixed-point theorem.[9]

Theorem (1): I will define E as a closed, convex, limited, and not-empty subset of the Banach space divided into sub-sets C .

Let $L: E \rightarrow E$ a constant was present. Mapping presupposes that there is a consistent. $Z \in [0, 1]$ thus, $u(F(X)) \leq kw(X)$ before every non-empty sub-group of C that. Subsequently, L contains a fix-point within the set C . For any not empty bounding sub-set X of $BC(\mathbb{R}_+)$, $x \in X$, $T > 0$ and $\epsilon \geq 0$ let

$$w^T(x, \epsilon) = \sup\{|x(s) - x(s)| : t, s \in [0, T], |t - s| \leq \epsilon\}$$

$$w^T(x, \epsilon) = \sup\{u^T(x, \epsilon) : x \in X\},$$

$$w_0^T(x) = \lim_{\epsilon \rightarrow \infty} u_0^T(x, \epsilon),$$

$$w_0^T(x) = \lim_{T \rightarrow \infty} u_0^T(x),$$

$$X(t) = x(t) : \text{the variables } x \in X\},$$

$$\text{diam the function } X(t) = \sup\{|x(t) - y(t)| : x, y \in X$$

And

$$\mu(X) = w_0(x) + \lim_{T \rightarrow \infty} \sup \text{diam the function } X(t) \quad (6)$$

Banas has demonstrated in [43] that the function μ measures non-compactness via space. $BE(\mathcal{R}_+)$. The solution to the equation as operative from the $BE(\mathcal{R}_+)$ included in $BE(\mathcal{R}_+)$

$$(F x)(t) = x(t) \quad (7)$$

We will discuss the introduction: the attractiveness to Eq. (7).

Definition 2: [9] If a ball $B(x_0, r)$ exists in space, then solutions to Eq. (7) are locally attractive. $BC(\mathbb{R}_+)$ so that for Any two solutions that are arbitrary to Eq. (7) that are part of $B(x_0, r) \cap \Omega$ that is in our possession.

$$\lim_{t \rightarrow \infty} (x(t) - y(t)) = 0 \quad (8)$$

Resolving the Eq. (7) is locally alluring at a uniform rate (or, similarly, asymptotic stability) if the limit (4) is consistent concerning $B(x_0, r) \cap \Omega$,

Theorem 2: There is a minimum of one solution to Eq. (5) in $BC(\mathcal{R}_+)$ for all of (5)'s solutions that have uniform local attraction.

Proof: This section presents a summary of the proof necessary for the subsequent sections. Refer to [10] for additional information.

First, the authors designed operator K in [2]; hence, for any $x \in BC(\mathbb{R}_+)$

$$(Hx)(t) = f\left(t, x(\alpha(t)), \int_0^{\beta(t)} u(t, s, x(\gamma(s))) ds\right),$$

(Hx) is clearly continuous on $\mathbb{R}_+$. Next, for each arbitrarily fixed $t \in \mathbb{R}_+$

$$|(Mx)(t)| \leq n |x(\alpha(t))| + M_0,$$

Where $M_0 = \sup\{|f(t, 0, 0)| : t \in \mathbb{R}_+\} + \Psi(2D)$,

$$r = \frac{M_0}{1 - n}.$$

Subsequently, they demonstrated that given A set that is not vacant $X \subset (Br)$, $\mu(HX) < n\mu(X)$, with $n \in U(0,1)$. Accordingly, the Eq. (1) of functional integration is at least one solution in British Columbia. $(\mathcal{R}_+)$, and (x) has a fixed- -point in (Br) for the operator H . based on Theorem 1. Every solution to Eq. (5) contained within the A member of the $\text{Ker } \mu$ group is ball Br

Corollary 1. If $f(t, s, 0)$ is constrained by additional Constraints 1–4, then the solutions to the integration of the formula (5) are as follows: are generally attractive, as stated in Theory 2. Sufficient evidence. See [27].

3- Cubic B-Aspline Scaling and Wavelet Function on [0.1]

In $L^2(\mathbb{R})$, you can utilize scaling functions to increase the size of any function. Extending these functions outside the integration domain is possible because they are specified across the natural line. This article considers B-spline scaling functions with compact support built for the bounded interval $[0,1]$. When using order m semi-orthogonal B-spline scaling functions, the requirement.

$$2^{J_0} \geq g$$

Has to be met for there to be one full inner scaling function. $\omega_{4,k}^{(j)}(x)$ represents these scaling functions. We'll employ scaling functions for cubic B-splines (cardinal B-splines of order $g = 4$). As a cubic spline, its scaling is denoted.

$$A_4(x) = \omega_4(x) = \begin{cases} \frac{1}{6}x^3 & 0 \leq x < 1 \\ \frac{1}{6}(-3x^3 + 12x^2 - 12x + 24) & 1 \leq x < 2 \\ \frac{1}{6}(3x^3 - 12x^2 + 60x - 44) & 2 \leq x < 3 \\ \frac{1}{6}(4-x)^3 & 3 \leq x < 4 \\ 0 & \text{otherwise} \end{cases}$$

Furthermore, the shape of its two-scale relation is

$$\omega_4(x) = \frac{1}{8}\omega_4(2x) + \frac{4}{8}\omega_4(2x-1) + \frac{6}{8}\omega_4(2x-2) + \frac{4}{8}\omega_4(2x-3) + \frac{1}{8}\omega_4(2x-4)$$

$$\omega_{4,-3}^{(3)}(x) = \begin{cases} (1-8x)^3 & 0 \leq x < \frac{1}{8} \\ 0 & \text{otherwise} \end{cases}$$

With boundary scale, for example, the scaling factors used for $j_0 = j = 3$ and $m = 4$ are enumerated below.

$$\omega_{4,-2}^{(3)}(x) = \begin{cases} 896x^3 - 288x^2 + 24x & 0 \leq x < \frac{1}{8} \\ 2(1-4x)^3 & \frac{1}{8} \leq x < \frac{2}{8} \\ 0 & \text{otherwise} \end{cases}$$

$$\omega_{4,-1}^{(3)}(x) = \begin{cases} -\frac{1408}{3}x^3 + 96x^2 & 0 \leq x < \frac{1}{8} \\ \frac{896}{3}x^3 - \frac{576}{3}x^2 + 36x - \frac{3}{2} & \frac{1}{8} \leq x < \frac{2}{8} \\ -\frac{1}{6}(4x-3)^3 & \frac{2}{8} \leq x < \frac{3}{8} \\ 0 & \text{otherwise} \end{cases}$$

$$\omega_{4,-1}^{(j)}(x) = \omega_{4,k}^{(3)}(2^{j-3}x), k = -3, -2, -1 \quad j = 3, 4, \dots$$

$$\omega_{4,5}^{(3)}(x) = \omega_{4,-1}^{(3)}(1-x),$$

$$\omega_{4,6}^{(3)}(x) = \omega_{4,-2}^{(3)}(1-x)$$

$$\omega_{4,7}^{(3)}(x) = \omega_{4,-3}^{(3)}(1-x)$$

$$\omega_{4,k}^{(j)}(x) = \omega_{4,2^j-k-4}^{(3)}(1-2^{j-3}x), \quad k = 2^j - 3, \dots, 2^j - 1, \quad j = 3, 4, \dots$$

Inner scaling:

$$\omega_{4,0}^{(3)}(x) = \begin{cases} \frac{256}{3}x^3 & 0 \leq x < \frac{1}{8} \\ -256x^3 - 128x^2 - 16x + \frac{2}{3} & \frac{1}{8} \leq x < \frac{2}{8} \\ 256x^3 - 256x^2 + 80x - \frac{22}{3} & \frac{2}{8} \leq x < \frac{3}{8} \\ \frac{32}{3}(1-2x)^3 & \frac{3}{8} \leq x < \frac{1}{2} \\ 0 & \text{Otherwise} \end{cases}$$

$$\omega_{4,1}^{(3)}(x) = \begin{cases} \frac{1}{6}(8x-1)^3 & \frac{1}{8} \leq x < \frac{2}{8} \\ -256x^3 + 224x^2 - 60x + \frac{31}{6} & \frac{2}{8} \leq x < \frac{3}{8} \\ 256x^3 - 352x^2 + 156x - \frac{131}{6} & \frac{3}{8} \leq x < \frac{1}{2} \\ \frac{1}{6}(5-8x)^3 & \frac{1}{2} \leq x < \frac{5}{8} \\ 0 & \text{Otherwise} \end{cases}$$

$$\omega_{4,2}^{(3)}(x) = \begin{cases} \frac{4}{3}(4x-1)^3 & \frac{2}{8} \leq x < \frac{3}{8} \\ -256x^3 + 320x^2 - 128x + \frac{50}{3} & \frac{3}{8} \leq x < \frac{1}{2} \\ 256x^3 - 448x^2 + 256x - \frac{142}{3} & \frac{1}{2} \leq x < \frac{5}{8} \\ \frac{1}{6}(6-8x)^3 & \frac{5}{8} \leq x < \frac{3}{4} \\ 0 & \text{Otherwise} \end{cases}$$

$$\omega_{4,3}^{(3)}(x) = \begin{cases} \frac{1}{6}(8x-3)^3 & \frac{3}{8} \leq x < \frac{1}{2} \\ -256x^3 + 416x^2 - 220x + \frac{229}{6} & \frac{1}{2} \leq x < \frac{5}{8} \\ 256x^3 - 544x^2 + 380x - \frac{521}{6} & \frac{5}{8} \leq x < \frac{3}{4} \\ \frac{1}{6}(7-8x)^3 & \frac{3}{4} \leq x < \frac{7}{8} \\ 0 & \text{Otherwise} \end{cases}$$

$$\omega_{4,4}^{(3)}(x) = \begin{cases} \frac{1}{6}(8x-4)^3 & \frac{1}{2} \leq x < \frac{5}{8} \\ -256x^3 + 416x^2 - 220x + \frac{229}{6} & \frac{5}{8} \leq x < \frac{3}{4} \\ 256x^3 - 544x^2 + 380x - \frac{521}{6} & \frac{3}{4} \leq x < \frac{7}{8} \\ \frac{1}{6}(7-8x)^3 & \frac{7}{8} \leq x < 1 \\ 0 & \text{Otherwise} \end{cases}$$

$$\omega_{4,k}^{(j)}(x) = \omega_{4,k}^{(3)}(2^{j-3}x - k), \quad k = 0, 1, \dots, 2^j - 4 \quad j = 3, 4, \dots$$

$$\begin{aligned} \psi_4(x) = & \frac{1}{8!} \omega_4(2x) + \frac{124}{8!} \omega_4(2x - 1) + \frac{1677}{8!} \omega_4(2x - 2) + \frac{7904}{8!} \omega_4(2x - 3) \\ & + \frac{18482}{8!} \omega_4(2x - 4) - \frac{24264}{8!} \omega_4(2x - 5) + \frac{18482}{8!} \omega_4(2x - 6) \\ & - \frac{7904}{8!} \omega_4(2x - 7) + \frac{1677}{8!} \omega_4(2x - 8) - \frac{124}{8!} \omega_4(2x - 9) \\ & + \frac{1}{8!} \omega_4(2x - 10) \end{aligned}$$

Cubic B-spline wavelet $\psi_4(x)$ is shown in Fig.3. The system's inner and border wavelet analysis is obtained through the application of [8, 11].

4. Functional Approximation

A function $f(x)$ specified in the interval $[0, 1]$ can be rendered in the cubic B-spline scale field FJ_0

For any fixed positive integer J_0 as

$$g(x) = \sum_{i=-3}^{2^{j_0}-1} c_{j_0,i} \varphi_{4,i}^{(3)}(x) + \sum_{i=3}^{j_0} \sum_{k=-3}^{2^i-4} d_{i,k} \psi_{4,k}^{(i)}(x) = C^T \Psi(x) \quad (9)$$

Where $\varphi_{4,i}^{(j_0)}$ and $\psi_{4,k}^{(i)}$ are wavelet and scaling functions should eq (9)'s infinite series be shortened, respectively. And for $j=3$, it can be expressed as follows:

$$g(x) = \sum_{i=-3}^7 c_{j_0,i} \varphi_{4,i}^{(j_0)}(x) + \sum_{i=3}^{j_0} \sum_{k=-3}^{2^i-4} d_{i,k} \psi_{4,k}^{(i)}(x) = C^T \Psi(x) \quad (10)$$

Where C and $\Psi(x)$ are $(2^{j_0+1} + 5) \times 1$ vectors given by

$$C = [c_{-3}, \dots, c_7, d_{3,-3}, \dots, d_{3,4}, \dots, d_{j_0,-3}, \dots, d_{j_0,2^j-4}]^T \quad (11)$$

$$\Psi = [\varphi_{4,-3}^{(3)}, \dots, \varphi_{4,-3}^{(3)}, \psi_{4,-3}^{(3)}, \dots, \psi_{4,-3}^{(3)}, \dots, \psi_{4,4}^{(3)}, \dots, \psi_{j_0,2^j-3}^{(3)}]^T \quad (12)$$

With

$$C_1 = \int_0^1 f(x) \varphi_{4,1}^{(3)}(x) dx, \dots, i = -3, \dots, 7. \quad (13)$$

$$d_{j,k} = \int_0^1 f(x) \psi_{4,1}^{(j)}(x) dx, \dots, j = 3, \dots, J_0, k = -3, \dots, 2^j - 4, \quad (14)$$

Where $\varphi_{4,1}^{(3)}$ and $\psi_{4,k}^{(j)}$ have dual purposes. of $\varphi_{4,1}^{(3)} i = -3, \dots, 7$ and $\psi_{4,k}^{(j)} = j = 3, \dots, J_0$ According to. By using linear combinations, these can be obtained. Of

$$\varphi_{4,1}^{(3)} \text{ and } \psi_{4,k}^{(j)} \quad (15)$$

$$\varphi = [\varphi_{4,-3}^{(3)}(x), \varphi_{4,-2}^{(3)}(x), \dots, \varphi_{4,-7}^{(3)}(x)]^T$$

$$\Psi = [\psi_{4,-3}^{(3)}(x), \psi_{4,4}^{(3)}(x), \dots, \psi_{4,2^j-4}^{(3)}(x)]^T \quad (16)$$

Using(9)-(13),(15,16) We obtain

$$\int_0^1 \phi(x)\phi^T(x) dx = L_1 \quad (17)$$

$\frac{1}{56}$	$\frac{7}{640}$	$\frac{31}{13440}$	$\frac{1}{566720}$	0	0	0	0	0	0	0
$\frac{7}{640}$	$\frac{31}{1120}$	$\frac{5}{256}$	$\frac{29}{6720}$	$\frac{1}{26880}$	0	0	0	0	0	0
$\frac{31}{13440}$	$\frac{5}{256}$	$\frac{183}{4480}$	$\frac{283}{10080}$	$\frac{239}{80640}$	$\frac{1}{40320}$	0	0	0	0	0
$\frac{1}{6720}$	$\frac{29}{6720}$	$\frac{283}{10080}$	$\frac{151}{2520}$	$\frac{397}{13440}$	$\frac{2}{672}$	$\frac{1}{40320}$	0	0	0	0
0	$\frac{1}{26880}$	$\frac{239}{80640}$	$\frac{397}{13440}$	$\frac{151}{2520}$	$\frac{397}{13440}$	$\frac{2}{672}$	$\frac{1}{40320}$	0	0	0
0	0	$\frac{1}{40320}$	$\frac{2}{672}$	$\frac{397}{13440}$	$\frac{151}{2520}$	$\frac{397}{13440}$	$\frac{2}{672}$	$\frac{1}{40320}$	0	0
0	0	0	$\frac{1}{40320}$	$\frac{2}{672}$	$\frac{397}{13440}$	$\frac{151}{2520}$	$\frac{283}{10080}$	$\frac{29}{6720}$	$\frac{1}{6720}$	0
0	0	0	0	$\frac{1}{40320}$	$\frac{2}{672}$	$\frac{397}{13440}$	$\frac{151}{2520}$	$\frac{283}{10080}$	$\frac{29}{6720}$	$\frac{1}{6720}$
0	0	0	0	0	$\frac{1}{40320}$	$\frac{239}{80640}$	$\frac{283}{10080}$	$\frac{183}{4480}$	$\frac{5}{256}$	$\frac{31}{13440}$
0	0	0	0	0	0	$\frac{1}{26880}$	$\frac{29}{6720}$	$\frac{5}{256}$	$\frac{31}{1120}$	$\frac{7}{640}$
0	0	0	0	0	0	0	$\frac{1}{6720}$	$\frac{31}{13440}$	$\frac{7}{640}$	$\frac{1}{56}$

L is $[11 \times 11]$.

Assume $\phi(x)$ serves dual functions. Of $\phi(x)$ as presented in the Equation

$$\varphi = [\varphi_{4,-3}^{(3)}(x), \varphi_{4,-2}^{(3)}(x), \dots, \varphi_{4,-7}^{(3)}(x)]^T$$

Using (8) (9) (11)

$$\int_0^1 \phi(x)\phi^T(x) dx = K11$$

where the identity matrix is $[11 \times 11]$. Consequently, we obtain

$$\bar{\omega} = \phi^{-1}\omega \quad (18)$$

Theorem 3: Let $e_j(x)$ denote the approximate errors of f in (9) using cubic B-spline scaling function within space V_j ; therefore, $\|e_j\| = O(2^{-4j})$.

Proof. By using (9) and (10). We get

$$e_j(x) = \sum_{i=j}^{\infty} \sum_{k=-3}^{2^i-4} d_{i,k} \psi_{4,k}^{(i)}(x)$$

By putting

$$c_j = \max\{|\psi^1(x)|; k = -3, \dots, 2^i - 4\} \quad \text{We obtain}$$

And

$$\sum_{k=-3}^{2^i-4} |d_{i,k} \psi_{4,k}^{(i)}(x)| \leq \alpha \beta c_i \frac{2^{-4!}}{4!}$$

As a result,

$$|e_j(x)| \leq \frac{1}{4!} \alpha \beta \sum_{i=j}^{\infty} c_i 2^{-4!}$$

The current inequality allows us to get

$$|e_j(x)| = O(2^{-4!}) \quad (19)$$

The order of error depends on the level j . as ()demonstrates. The approximation error will decrease with increasing degree of j .

5. Method of Solvation

This part outlines our primary approach, which combines the fixed point with the cub-spline scale function. Next, we consider the method's convergence.

5.1 The New Numerical Method's Description: When considering the integral Eq. (5). To streamline this procedure, assume that each value of t is confined to the interval where the maximum level of $\beta(t)$ is constant. assume $t \in [0, a]$ without losing generality. Allow me to

$$0 = t_0 \leq t_1 \leq t_m = a.,$$

By G locations in $[0, a]$.

That are evenly separated. By the proof, K is a continuous operator on (Br) and had a fix-point x in the Br —theorem 2. Eq. (1) has at least one solution in Br under the assumptions 1–4. Additionally, evenly locally appealing solutions for problems (1). This section provides a concise overview of the evidence required in the subsequent sections. For additional information, please refer to [6].

Initially, parameter G was defined by the creators in [6]. in a way that ensures that for any $x \in Br$,

$$x(t) = (Kx)(t) = f \cdot \left(t, x(\alpha(t)), \int_0^{\beta(t)} u(t, s, x(\gamma(s))) ds \right),$$

Now, we treat operator H using the fix-point approach. for $x_0(t) \in Br$ as well as Points t_i ($i = 1, \dots, G$)

$$x_{k+1}(t_i) = (H_k)(t_i) = f \cdot \left(t_i, x_k(\alpha(t)), \int_0^{\beta(t)} u(t_i, s, x_k(\gamma(s))) ds \right) \quad K = 0, 1, \dots \quad (20)$$

The integral is to be approximated. Numerically inside the intervals $[0, \beta(t_i)]$ in (21) and $x_{N+1}(t)$, we apply A Simpson rule that is composite and applies to equally distant L points. Today, we utilize cub-spline scaling methods as the foundation for our estimations. $x_{N+1}(t)$ to get ready for the following iteration. $x_{N+1}(t)$ We can immediately calculate using the coefficients of the scaling functions (5)–(14) from the previous section without having to solve any systems of algebraic equations as

$$z_i = \int_0^1 x_{k+1}(t) \varpi_{4,i}^{(3)}(t) dt, \quad i = -3, \dots, 7$$

where, as previously stated

$$[\varpi_{4,-3}^{(3)}(t), \varpi_{4,-2}^{(3)}(t), \dots, \varpi_{4,7}^{(3)}(t)]^T = P^{-1} [\omega_{4,-3}^{(3)}(t), \dots, \omega_{4,7}^{(3)}(t)]^T$$

Given the values of $x_{k+1}(t_i)$ ($i = 1, \dots, H$), we compute c_j . By employing the composite Simpson rule, we arrive at the following;

$$x_{k+1}(t) \approx \sum_{i=-3}^7 c_i \varphi_{4,i}^{(3)}(t) \quad (21)$$

We repeat the iterations until the difference between subsequent iterations, $x_k(t)$, is as small as we need for the appropriate level of precision. The end values of $x_{k+1}(t)$ correspond to an operator's M fixed point k at that level of precision. Consequently, we make an approximation of Equation (5). The following briefly describes the numerical approach.

5.2 There is a Relationship Between Teachers K and G: The teacher k represents the number of iterations within the fixed-point method, while G, according to the assumptions of Theory (2), are distant and central points within the interval $[0, a]$. Through the practical application of numerical examples, we continue with the iterations until we achieve a small difference between consecutive iterations. These small differences are essential for achieving high accuracy, ensuring that the terminal parameters $x(t)$ converge to a stationary point for operator Z. Therefore, we are approaching an accurate solution to the Eq. (5). We observe the accuracy of the method used through the numerical examples in examples [7] and [8], comparing them with the methods used in previous studies. We notice that an increase in k and G leads to an increase in accuracy and a decrease in the absolute error rate. The iterations keep increasing until the approximate solutions get closer to the exact solutions.

6- Results:

This article illustrates the method's accuracy by presenting numerical illustrations for the integral equation eq(5). We utilized the symbol k to represent the number of iterations in the fixed-point method. And K to Represent the approximate value $x(t)$ in the iteration $X(t)$, which is based. On this, we can calculate the absolute error ratio to $x(t)$ in iteration k as follows: is represented with $x_{K(t_i)}$

$$|(X(t)) - x_{K+1}(t)|$$

Hence, it is possible to determine the most significant (absolute error in iteration K as an

$$\|x - x_K\| = \max |x(t_i) - x_{K(t_i)}|$$

Furthermore, one can compute the discrepancy located between the approximate at K and $K+1$ as

$$|x_{K+1}(t) - x_K(t)|.$$

As a result, we acquire

$$\|x - x_K\| = \max |x_{K+1}(t) - x_K(t_i)|$$

We used different values of k and G to solve the following numerical examples. In these examples, we applied the formulas in the articles above (17) and (18) to derive approximate numerical solutions. The calculations and results were carried out using Mathematics 8.

Example (1) : non-linear Fredholm–Hammerstein equation that follows

$$x(t) = \sin\left(\frac{\pi}{2}t\right) - 2te^{-t}\ln(3) + e^{-t} \int_0^1 \frac{4ts + \pi t \sin(\pi s)}{x(s)^2 + s^2 + 1} ds,$$

Table 1: Absolute errors for Example 1 across various values of K .

t	k=2	k=5	k=10
0	0	0	0
0.1	0.127890×10^{-1}	0.953012×10^{-5}	0.362567×10^{-9}
0.2	0.214093×10^{-1}	0.167951×10^{-4}	0.119137×10^{-8}
0.3	0.227909×10^{-1}	0.197670×10^{-4}	0.139588×10^{-7}
0.4	0.169766×10^{-1}	0.149812×10^{-4}	0.402300×10^{-7}
0.5	0.797177×10^{-2}	0.497886×10^{-4}	0.934621×10^{-7}
0.6	0.384727×10^{-4}	0.334333×10^{-5}	0.378213×10^{-7}
0.7	0.472688×10^{-2}	0.660849×10^{-5}	0.269398×10^{-7}
0.8	0.631157×10^{-2}	0.612657×10^{-5}	0.301620×10^{-7}
0.9	0.572703×10^{-2}	0.453392×10^{-5}	0.464820×10^{-7}
1	0.410323×10^{-2}	0.290023×10^{-5}	0.301723×10^{-8}

Possesses an exclusive, precise resolution

. $x(t) = \sin\left(\frac{\pi}{2}t\right)$. Functional, the value of $\beta(t)$ is 1, and $\gamma(t)$ is equal to t .

$$f(t, x, y) = \sin\left(\frac{\pi}{2}t\right) - 2te^{-t}\ln(3) + e^{-t}y$$

And

$$u(t, s, x) \frac{4ts + \pi t \sin(\pi s)}{x^2 + s^2 + 1},$$

Are continuous functions that meet Theorem 2's presumptions. see

$$\phi(t) \text{ Equal } t, k \text{ Equals } 0, m(t) = e^{-t}, \text{ where } D = 1.24575$$

We make a decision $x_0(t) = \sin\left(\frac{\pi}{2}t\right) - 2te^{-t}\ln(3) \in [-r, r]$, where $r = \frac{M_0}{1-n} = 3.82984$.

Table 1 displays the absolute values of defects for ($G = 200$ and $L = 200$) mesh points. The errors associated with a single iteration are represented by absolute values in **Table 2**. for varied numbers of mesh points. Furthermore, errors are diminished by conducting additional iterations. Refer to Figure 1. Where the $[\log_{10}(\|x_k - x\|)] = \log_{10}(\max |x_k(t_i) - x(t_i)|)$, to $t_i = i/10$ for any ($i = 0, 1, \dots, 10$)

Example (2): Examine the subsequent information. Kind (42, 43) of non-linear function integral equation of the Volterra.

$$x(t) = \frac{t}{1+t^2} x(t) + \int_0^t e^{-t} \frac{sx(s)}{1+|x(s)|} ds$$

Table 2: Absolute Errors within the EX.1 K Equal 10, L Equal 200, many G variables

t	G=50	G=100	G=200
0	0	0	0
0.1	0.170303×10^{-5}	0.449863×10^{-5}	0.362567×10^{-9}
0.2	0.307753×10^{-5}	0.805820×10^{-5}	0.119137×10^{-8}
0.3	0.409215×10^{-5}	0.104431×10^{-4}	0.139588×10^{-7}
0.4	0.478569×10^{-5}	0.117460×10^{-4}	0.402300×10^{-7}
0.5	0.521326×10^{-4}	0.121565×10^{-4}	0.934621×10^{-7}
0.6	0.540250×10^{-4}	0.120009×10^{-4}	0.378213×10^{-7}
0.7	0.548649×10^{-4}	0.116624×10^{-4}	0.269398×10^{-7}
0.8	0.547760×10^{-4}	0.110637×10^{-4}	0.301620×10^{-7}
0.9	0.542085×10^{-4}	0.105845×10^{-4}	0.464820×10^{-7}
1	0.532832×10^{-4}	0.101492×10^{-4}	0.301723×10^{-7}

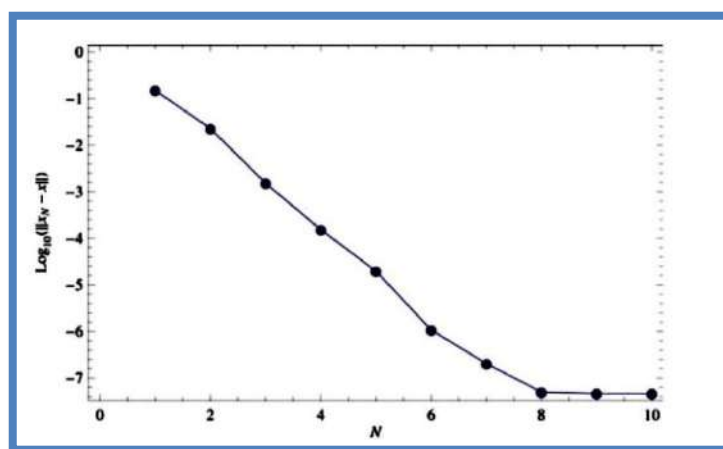


Figure 1: Ex(1) Illustrates the Logarithm of t the Utmost Error Occurring During Every Iteration.

Which has a unique, precise solution. The value of $x(t)$ is zero. The formulas are: $\alpha(t) = \beta(t) = \gamma(t) = t$: We can ascertain t by employing fundamental mathematics. the solution to which is unique and exact $x(t) = 0$. The functions $\alpha(t)$, $\beta(t)$, and $\gamma(t)$ are all defined as t .

$$f(t, s, x) = \frac{tx}{1+t^2} + y,$$

And

$$u(t, s, x) = e^{-t} \frac{sx}{1+|x|}$$

Satisfying the requirements of the second theorem with $\phi(t)$ Equal t , k Equal $1/2$, $m(t)$ Equal 1 , and $D = 0.270671$, The procedure is implemented. where $x_0(t)$ Equal $0.5 \in Br$, where $r = 1.08268$. **Table 3** displays the exact error values, while Figure 2 depicts the logarithm of the highest error associated with iterations, especially for G equals 80 and L equals 50 mesh points.

Example (3): We investigate the non-linear Volterra functional integral problem using proportional delay.

$$x(t) = \cos(t) - \sin\left(\frac{e^{-\cos^2(\frac{1}{2})} - e^{-1}}{1+t^2}\right) + \sin\left(\int_0^{t/2} \frac{\sin(2s)e^{-x^2(s)}}{1+t^2} ds\right),$$

Table3: Fundamental errors for Example 2 over several levels of K

t	k=5	k=25	k=50
0	0	0	0
0.1	0.462407×10^{-5}	0.808969×10^{-13}	0.306027×10^{-20}
0.2	0.131506×10^{-3}	0.268324×10^{-12}	0.107748×10^{-19}
0.3	0.790865×10^{-3}	0.275392×10^{-12}	0.171832×10^{-19}
0.4	0.243661×10^{-2}	0.300346×10^{-12}	0.114517×10^{-19}
0.5	0.512032×10^{-2}	0.649454×10^{-10}	0.156528×10^{-18}
0.6	0.0835707×10^{-2}	0.635856×10^{-9}	0.148931×10^{-17}
0.7	0.114419×10^{-1}	0.316263×10^{-8}	0.185272×10^{-16}
0.8	0.138099×10^{-1}	0.802217×10^{-8}	0.131274×10^{-15}
0.9	0.151984×10^{-1}	0.129801×10^{-7}	0.334904×10^{-15}
1	0.156251×10^{-1}	0.149012×10^{-7}	0.439281×10^{-15}

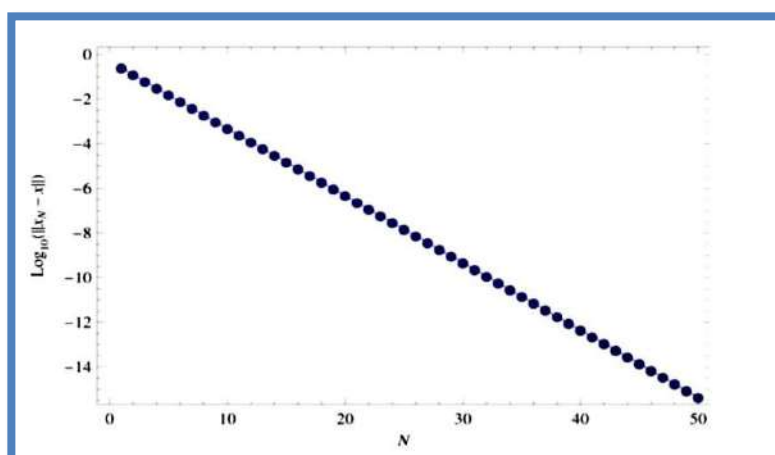


Figure 2: The logarithm of the highest error associated with every iteration in Example 2.

Which possesses the precise solutions $x(t) = \cos(t)$. The functions such as $\beta(t) = t - (1 - q)t$ for $q = 1/2$ and $\gamma(t)$ Equal t can be readily demonstrated to be continuous. Additionally, $\alpha(t)$ may be Any continuous operation that satisfies Condition 1.

$$f(t,x,y)=\cos(t)-\sin\left(\frac{e^{-\cos^2(\frac{1}{2})}-e^{-1}}{1+t^2}\right)+\sin(y)$$

And
$$u(t,s,x)=\cos(t)-\sin\left(\frac{e^{-x^2}\sin(2x)}{1+t^2}\right)$$

Meet the conditions of Theorem 2, given $k = 0$, $m(t) = 1$, $\phi(t) = t$, with D ranging from 0.25. We

Derive $r = 1.5$ and utilize $x_0(t) = \cos(t) - \sin\left(\frac{e^{-\cos^2(\frac{1}{2})}-e^{-1}}{1+t^2}\right) \in Br$

The preliminary function. Both **Table 4** and the following figure illustrate the absolute errors with the logarithm of the highest Iteration-related absolute errors for $L = 100$ and $G = 200$ centered within the range $[0,2]$.

Table 4: Approximate Errors for the Third Example for Various Values of K

t	k=2	k=5
0	0	0
0.2	0.234328×10^{-7}	0.239081×10^{-7}
0.4	0.146720×10^{-7}	0.360304×10^{-7}
0.6	0.323011×10^{-6}	0.914190×10^{-7}
0.8	0.863026×10^{-6}	0.637923×10^{-9}
1	0.205252×10^{-5}	0.284243×10^{-9}
1.2	0.539092×10^{-5}	0.878520×10^{-7}
1.4	0.113864×10^{-4}	0.593672×10^{-7}
1.6	0.154702×10^{-4}	0.506919×10^{-7}
1.8	0.252727×10^{-4}	0.231660×10^{-7}
2	0.852385×10^{-7}	0.116951×10^{-8}

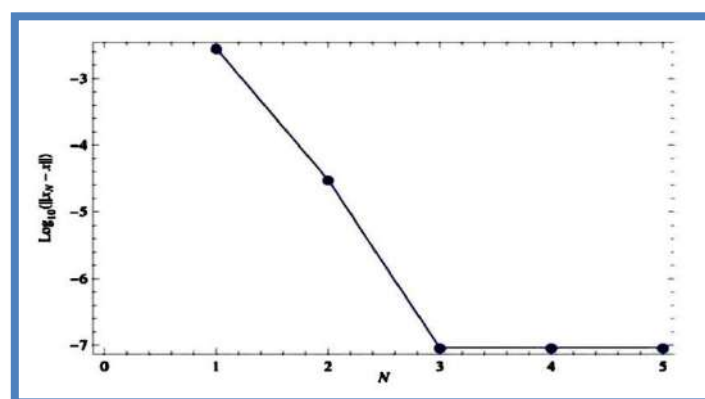


Figure 3. Logarithm of the greatest error associated with each repetition in Example (3).

Example (4): The subsequent non-linear integral Equation for Volterra function

$$x(t) = e^{-t\sqrt{t}} - \frac{t^2(1 + e^{-t\sqrt{t}}x(t))}{2(1 + t^4)} + \left(\frac{t^2}{1 + t^4}\right) e^{-\int_0^t \sqrt{s} \frac{2e^{-s}x(\frac{\sqrt[3]{s^2}}{2})ds}{(1+x(\frac{\sqrt[3]{s^2}}{2}))}}$$

Contains a precise solution. The value of $x(t)$ is equal to e . The functions to use are $\alpha(t) = t$, $\beta(t) = t$, and $\gamma(t)$ Equal 3.

Comply with the first condition of Theorem 2. Furthermore, functions as well

$$f(t, x, y) = e^{-t\sqrt{t}} - \frac{t^2(1 + e^{-t\sqrt{tx}})}{2(1 + t^4)} + \left(\frac{t^2}{1 + t^4}\right) e^{-|y|}$$

And
$$u(t, s, x) = \frac{2e^{-tx}}{1+x^2}$$

Fulfilled the criteria of Theorem 2–4 using $n = 0.111641$, $\phi(t)$ Equal t , $G(t) = t^2/(1+t^4)$, $D = 0.5$, and $M_0 = 2$. Thus, we have $r = 2.25134$ and choose $x_0(t) \in \text{Br}$. The extent of errors

Table 5: Absolute Errors of Example 4 for $x_0(t) = 0.5$ and Different Value of K

t	K=2	K=5	K=10
0	0	0	0
0.1	0.116062×10^{-4}	0.302756×10^{-6}	0.302763×10^{-6}
0.2	0.150179×10^{-3}	0.164254×10^{-6}	0.163322×10^{-6}
0.3	0.597392×10^{-3}	0.518220×10^{-6}	0.551065×10^{-6}
0.4	0.139726×10^{-2}	0.963558×10^{-6}	0.626378×10^{-6}
0.5	0.237648×10^{-2}	0.135418×10^{-5}	0.213892×10^{-6}
0.6	0.319742×10^{-2}	0.444057×10^{-5}	0.933230×10^{-7}
0.7	0.356863×10^{-2}	0.811830×10^{-5}	0.411882×10^{-7}
0.8	0.342076×10^{-2}	0.110323×10^{-4}	0.286523×10^{-6}
0.9	0.291232×10^{-2}	0.133351×10^{-4}	0.152142×10^{-6}
1	0.22864×10^{-2}	0.136016×10^{-4}	0.216699×10^{-6}

Table 6: Absolute errors of Example 4 for $K=5$ and different values of $x_0(t)$

t	$x_0(t)=0.5$	$x_0(t) = 1$	$x_0(t)=2$
0	0	0	0
0.1	0.302756×10^{-6}	0.302762×10^{-6}	0.302777×10^{-6}
0.2	0.164254×10^{-6}	0.163079×10^{-6}	0.161031×10^{-6}
0.3	0.518220×10^{-6}	0.562475×10^{-6}	0.637346×10^{-6}
0.4	0.963558×10^{-6}	0.420642×10^{-6}	0.419744×10^{-6}
0.5	0.135418×10^{-5}	0.165281×10^{-5}	0.595858×10^{-5}
0.6	0.444057×10^{-5}	0.553731×10^{-5}	0.188352×10^{-4}
0.7	0.811830×10^{-5}	0.140381×10^{-4}	0.417234×10^{-4}
0.8	0.110323×10^{-4}	0.251467×10^{-4}	0.680604×10^{-4}
0.9	0.133351×10^{-4}	0.133351×10^{-4}	0.880009×10^{-4}
1	0.136016×10^{-4}	0.397919×10^{-4}	0.991856×10^{-4}

Table 5 provides details of the iterations, which correspond to $G = 150$ and $L = 100$ mesh points. **Table 6** displays the maximum inaccuracy values for a single iteration according to various initial locations and a consistent number of mesh points. **Figure 4** illustrates how increasing the number of repetitions reduces mistakes.

Example (5): Examine the integral Equation shown below [10].

$$x(h) = \frac{\sin(hx(\sqrt[3]{h}))}{1+h^2} + \arctan\left(\int_0^{\sqrt{1}} \frac{\sqrt[4]{1+sx^2(s^2)} + hs^{11}(1+x^4(x^2))}{(1+h^7)(1+x^4(x^2))} ds\right).$$

Using basic math, we can determine that

$$\alpha(h) = \sqrt[3]{t}, \beta(h) = \sqrt{h}, y(t) = h^2$$

$$x(h) = \frac{\sin(hx(\sqrt[3]{h}))}{1+h^2}, +\arctan(y)$$

And

$$u(t, s, x) = \frac{\sqrt[4]{1 + sx^2(s^2) + ts^{11}(1 + x^4)}}{(1 + h^7)(1 + x^4)}$$

Fulfill the requirements of Theory 2 accompanied by $\{D = 1.0184, k = 0.5, m(t) = 1, \text{ and } \phi(t) = h.\}$ As a result, we determine that the beginning function belongs to the ball Br and get($r = 4.07362$.) In contrast, since $f(t, x, 0)$ is a function restricted in its range, Corollary 1's requirement are met. As a result, there is at least one solution to the integral Equation, and These solutions possess universal appeal. Upon completing 50 cycles, we ascertain the mistake for ($L = 50$ and G is equal to 80) mesh points. $\|x_{N+1} - x_N\| = \max_i |x_{N+1}(t_i) - x_N(t_i)| = 2.37155 \times 10^{-8}$, where $h_i = i/10$ where $i = 0, 1, \dots, 10$. In the figure, the response is an approximation obtained after 50 iterations. as demonstrated by the explanation that follows $x(t)$ is an element of Br

Example (6): Examine: Kindly assess the presented non-linear Fredholm integral. [44,45]

$$x(t) = 1 + te - \int_0^1 (t + S)e^{x(S)} dS.$$

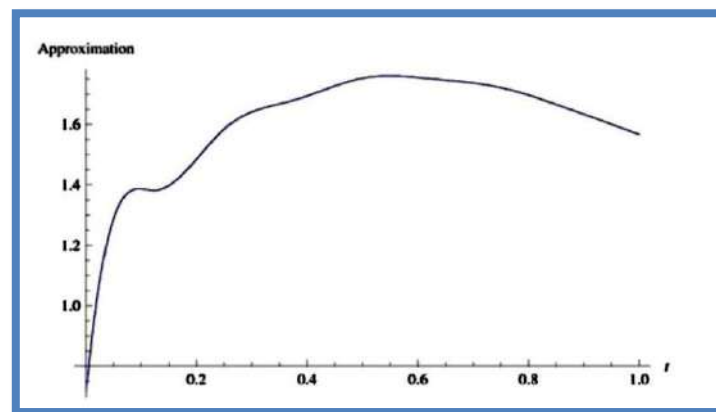


Figure 4: Approximation Solution of Example 5 after 50 Iteration

The precise solution is $x(t) = t$. This section addresses the solution of the integral Equation utilizing our suggested method and juxtaposes prior results with the new findings delineated in Table 7 for $K = 30$, $G = 200$, and $L = 200$.

Example (7): The following integral Equation

$$x(t) = \sin(t)^2 + 1 - \int_0^t 3 \sin(t - s)x(s)^2 ds$$

Table 7: As shown in Figure 3, the absolute errors are as follows: $K=30$, $G=200$, and $D=200$.

t	Methodology of [44]	Methodology [45]	Methodologically presented
0	2×10^{-3}	2.58×10^{-6}	8.96539×10^{-10}
0.2	1×10^{-2}	7.35×10^{-6}	7.99324×10^{-9}
0.4	2×10^{-2}	7.93×10^{-6}	4.23015×10^{-8}
0.6	1×10^{-2}	2.55×10^{-6}	8.28779×10^{-9}
0.8	0×10^{-3}	3.98×10^{-6}	2.82284×10^{-7}
1	1×10^{-2}	2.64×10^{-6}	5.62541×10^{-8}

Table 8: Contains The Solutions to Example 7: $K = 10$, $G = 250$, And $D = 200$, With Both Exact And Approximate Values

t	Methodology of [44]	Methodology of [45]	Methodologically presented	PERFECT RESOLUTION
0	1.0000	1.00000	1.000000000	1.000000000
0.1	0.9952	0.99500	0.995004165	0.995004165
0.2	0.9800	0.98006	0.980066580	0.980066577
0.3	0.9554	0.95533	0.955336485	0.955336489
0.4	0.9210	0.92105	0.921060993	0.921060994
0.5	0.8775	0.87756	0.877582565	0.877582561
0.6	0.8255	0.82531	0.825335614	0.825335614
0.7	0.7648	0.76482	0.764842185	0.764842187
0.8	0.6969	0.69669	0.696706707	0.696706709
0.9	0.6217	0.62159	0.621609968	0.621609968
1	0.5405	0.54028	0.540302303	0.540302305

The exact answer is $x(t)$ Equal $\cos(t)$. This Equation fails to satisfy the following criteria: to theory 2, we employ the technique described in this article. Furthermore, the findings of our investigation have been compared with the results [46,47] Table 8.

7. Conclusion

In this research paper, we applied fixed-point technique with cubic B-Spline scaling function to obtain a numerical solution for a set of non-linear integral equations without the need for algebraic systems. Using the numerical examples and the obtained results, as well as equation number (5), we found that the results are highly accurate and closely approximate the exact solution. We compared these results with those obtained from previous studies and observed the accuracy of this method. Since the accuracy of this method depends on using larger values for G and K , we note that the accuracy will improve with larger values. We continue the iterations until we reach a precise solution that approaches the true solution. This method is free from scenarios, problems, or excessive computational costs, and it is applicable to equations with larger and more complex dimensions. This method has no adverse effects on more complex equations.

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Assessment of IoMT-Based Remote Patient Monitoring Used to Support Healthcare System in Kirkuk City

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Keywords: Iomt, Iot, Healthcare, Remote Patient Monitoring, Telemedicine, Iot Cloud, Iot Platform, Reliability.

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Abstract:

Recently, Internet of Medical Things (IoMT)-based telemedicine applications are acquiring increasing attention. This is due to their benefits, especially during pandemic circumstances. A popular example is the Remote Patient Monitoring (RPM) system, whose performance crucially depends on the components used, mainly the available Internet connectivity. Since similar RPM systems operating in different areas can perform differently, the reliability of these systems is questionable. Therefore, in this paper, the reliability of an RPM system is assessed and tested under realistic operation conditions in Kirkuk city. The purpose is to figure out to what extent the RPM system is applicable under the locally available technologies. Extensive tests by using a 12 Mbps optical Internet connection and two different cloud platforms show that the system encountered a maximum delay of about 59 seconds with an average availability exceeding 98%. The tests proved the robustness of the system, and it is henceforth recommended for practical application in Kirkuk city to support the healthcare system.

Keywords: Iomt, Iot, Healthcare, Remote Patient Monitoring, Telemedicine, Iot Cloud, Iot Platform, Reliability.

تقييم نظام مراقبة المرضى عن بعد المعتمد على تقنية IoMT المستخدم لدعم نظام الرعاية الصحية في مدينة كركوك

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الخلاصة:

تحتل تطبيقات التطبيب عن بعد المرتكزة على شبكات إنترنت الأشياء الطبية (IoMT) باهتمام متزايد. وذلك لفوائدها خاصة في ظل ظروف انتشار الأمراض والأوبئة من الأمثلة الشائعة على ذلك نظام مراقبة المريض عن بعد (RPM) الذي يعتمد أدائه بشكل حاسم على المكونات المستخدمة، وخاصة جودة خدمة الاتصال بالإنترنت المتاحة وهذا يعني أن أنظمة RPM نفسها التي تعمل في مناطق مختلفة من العالم يمكن أن يكون مستوى أدائها متبايناً، مما يجعل من موثوقية هذه الأنظمة موضع شك. تم في هذا البحث تقييم واختبار موثوقية نظام RPM في ظل ظروف تشغيل واقعية في مدينة كركوك. ان الغرض من ذلك هو معرفة مدى إمكانية تطبيق نظام RPM في ظل التقنيات المتاحة محلياً. أظهرت الاختبارات المكثفة باستخدام اتصال إنترنت بصري بسرعة ١٢ ميجابايت في الثانية ومنصتين سحابيتين مختلفتين أن أقصى تاخير يبلغ حوالي ٥٩ ثانية و بمتوسط توفر (Availability) يتجاوز ٩٨٪. لقد أثبتت الاختبارات ان هذا النظام يتمتع باعتمادية عالية بالنسبة لتطبيقات مراقبة المرضى عن بعد ويمكن ان يدعم ويحسن نظام الرعاية الصحية في مدينة كركوك.

الكلمات المفتاحية: إنترنت الأشياء الطبية، إنترنت الأشياء، الرعاية الصحية، مراقبة المرضى عن بعد، التطبيب عن بعد، سحابة إنترنت الأشياء، منصة إنترنت الأشياء، الموثوقية.

1. Introduction

The Internet of Things (IoT) is a concept that encompasses the interaction between Internet-connected devices less need for human-to-human or human-to-computer communication. These devices can offer services globally [1], in ways that were previously unimaginable. In contrast to the conventional paradigm, all entities in IoT realm are regarded as intelligent objects that are interconnected [2]. However, IoT-based technologies have significantly contributed to the transformation of the healthcare sector into a technologically advanced domain, with the aim of enhancing patient care. The Internet of Medical Things (IoMT), is the result of applying IoT in healthcare. It combines different medical devices and sensors with the Internet to enable immediate gathering and examination of patient data [3].

The deployment of IoMT in the healthcare sector brings substantial advantages, such as the ability to provide effective and prompt medical assistance using precise and continuous monitoring data [1]. However, the concept of green hospitals evolving as a model for future hospitals [4], when equipped with IoMT technologies, can significantly enhance the healthcare

ecosystem, particularly for individuals utilizing remote monitoring systems [5]. Remote monitoring systems rely heavily on the processing and analysis of real-time data obtained by bio-sensors [2].

The core of IoMT-based healthcare systems is the IoT platform. It provides the required services and applications such as data transfer, storage, and processing. The Arduino IoT Cloud and Blynk are popular platforms for developing IoT-based applications in the healthcare sector. The Arduino Cloud is the standard and compatible platform for Arduino controllers and peripherals. Also, the effectiveness of the Blynk platform has been emphasized through its successful usage in many IoMT applications [6]. Both platforms are widely applied in remote health monitoring systems due to their flexibility, ease of use, and very good compatibility with different types of sensors and devices [7]. The quality and reliability of these services highly depend on many factors, including the Internet connectivity and the sensors and actuators [8],[9]. Then, IoMT-based healthcare systems may encounter serious challenges such as delay and data loss when they use unstable Internet connectivity, which is the case in many places.

However, the application of IoMT in Kirkuk city in Iraq can be very beneficial to the healthcare system in the city. As the population and hence the demand for healthcare services increases, the infrastructure of healthcare in the city is overstretched. Also, access to specialized care can become difficult for people from far-flung areas or living far away from the city center, and thus, on-time access to healthcare gets compromised. Therefore, this entails that an effective and reliable telehealth monitoring system becomes very important in helping to surmount such challenges by providing a system through which healthcare providers monitor the health status of their patients remotely. However, IoMT acquires more importance in cases like the spread of infectious diseases where it would be preferable to monitor patients remotely without having to visit health facilities and hospitals frequently except in a situation of critical need.

The objective of this study is to design, implement, and test the performance of a medical IoT-based remote patient monitoring system. In order to highlight the impact of the IoT platform on the performance of the system, the system is implemented and tested by using two popular IoT platforms, namely the Arduino IoT and Blynk platforms. Extensive tests under realistic operation conditions have been conducted. The system's real-time operation and reliability have been evaluated by focusing on several key numerical performance metrics such as maximum delay, maximum and average data loss, uptime, and availability. By conducting this evaluation, this study aims to show the significance of developing and deploying such a system in Kirkuk city.

The rest of the paper is organized as follows: A comprehensive literature review on the works related to the context of this paper is presented in Section 2. In Section 3, the IoMT system model is described in terms of implementation and workflow. Then, in Sections 4 and 5, the reliability metrics and the adopted test procedure are presented. Section 6 presents the results obtained from testing the system under realistic operation conditions. Reliability calculations and related comments are also given in Section 6. Finally, conclusions and recommendations are given in Section 7.

2. Literature Review

Many designs and implementations of Remote Patient Monitoring (RPM) that used different sensors and actuators were presented in the literature, such as [3],[10],[11], and [12]. They collectively showed the advantages of IoMT-based RPM including: The ability of healthcare providers to collect and evaluate patient data from a distance, decreasing the necessity for frequent visits to healthcare institutions, real-time collection and updating of health data and triggering alerts and notifications to healthcare practitioners enabling them to stay informed about the health status of their patients even when they are not physically present in the hospital.

However, the performance of IoMT RPM greatly relies on the capabilities of the used communication infrastructure. Therefore, these reported advantages of IoMT cannot be directly generalized for all implementations of IoMT [13]. One main reason is that the performance of the RPM system is governed by the available Internet connectivity. Slow service can significantly cause undesired delays and data losses, even if the IoT platform is working properly. Such cases may cause serious hazards to patients' lives. Therefore, it is important to test and evaluate the reliability of the implemented RPM system for the specific components used to build the system, to know to what extent this system is dependable [14].

Reliability is essential in healthcare operations as it ensures that systems can effectively manage and counteract external stresses, such as patient demand and resource availability. A reliable healthcare system contributes to performance enhancement and customer satisfaction, which are critical in delivering quality care [15]. Beyond life-threatening issues, unreliable systems can result in significant financial losses for healthcare providers. This can affect the overall reputation of healthcare institutions, making reliability a key factor in operational success. The increasing demand for new technologies necessitates robust reliability measures to ensure these systems can handle various scenarios without failure [16]. Healthcare systems are complex and high-risk, involving various components that must work together reliably. This complexity necessitates a thorough reliability analysis to identify weak links and potential failure points. Reliability analysis allows for the identification of weak links within the system,

enabling targeted improvements. This process helps in redesigning systems to enhance reliability and reduce the likelihood of failures [17].

The study [18] emphasizes the reliability of IoMT-based biomedical measurement systems (BMS) through accurate measurements essential for diagnosing diseases like heart conditions and neurological disorders. Validation against standard methods shows promising results, with low mean absolute differences in blood sugar measurements. Advanced signal processing techniques significantly improve data quality, enhancing the reliability of ECG signal classification. Addressing environmental challenges and employing multidisciplinary approaches are crucial for maintaining consistent performance in real-world applications.

In [19], the importance of reliability in telemedicine systems was studied for real-time remote health monitoring. It proposed solutions to mitigate electromagnetic interference and packet delay. The study reported the need for secure, efficient, and effective communication systems to support healthcare delivery. Next, a complete design and implementation of a wireless clinical monitoring system was presented in [20] for measuring pulse and oxygen saturation from patients. A clinical trial was conducted over seven months in a cardiology unit with 41 patients. The system demonstrated high reliability with an average availability of 99.68%. However, the reliability of the sensors for measuring oxygen and pulse was lower at 80.85%, with occasional disconnections. The performance was improved by increasing the sampling rate and implementing a disconnection alarm system. The analysis showed that the system was effective in detecting clinical deterioration. The results indicate the feasibility of using wireless networks for patient monitoring in hospitals [17]. The study [21] presents a comprehensive analysis of IoMT applications, highlighting their reliability through systematic methodology, diverse applications, and technological support, while also addressing potential challenges that could affect their implementation in healthcare systems.

An experimental performance test was presented in [22]. The presented tests were conducted on what is called Health Monitoring for All (HM4All) with a remote vital signs monitoring system based on a ZigBee body sensor network (BSN). The system involved six ECG sensors operating in two modes: ECG mode, which transmits ECG waveform data and heart rate (HR) values, and HR mode, which only transmits HR values. The non-beacon-enabled star network maintained a 100% delivery ratio (DR) without hidden nodes. When the network topology was changed to a 2-hop tree, the performance slightly degraded, resulting in an average DR of 98.56%. However, further investigation revealed that individual sensing devices experienced transitory periods with low DR.

The study in [23] aimed to assess the reliability of a real-time health monitoring system in the homes of older adults. The "Mobile Care Monitor" system was installed in nine homes for two weeks, featuring a wireless wristwatch with sensors and additional devices. Results showed system reliability between 73% and 92%, with data concurrence exceeding 88%. Usability metrics ranged from 82% to 97% after a firmware update for the pulse oximeter. Watch-wearing adherence was about 80%, and the study achieved 88% effectiveness in collecting potential measurements, indicating the system's high effectiveness in providing accurate remote health data.

In [24], the regional limitations were taken into consideration. The study evaluated what is called the "Mashavu" telemedicine system in rural Kenya. It was found that it provided about 90% of the same medical advice as face-to-face consultations, with a high level of diagnostic consistency. The system was capable of enhancing healthcare access, reducing travel costs, and creating job opportunities. At the same time, it faced limitations such as potential bias and diagnostic gaps. Future improvements were suggested to focus on quality control and expanding diagnostic capabilities.

However, COVID-19 has shown the significance of IoMT-based RPM systems. In [25], an IoT-based health monitoring system was presented that is designed for individuals diagnosed with COVID-19. It created a prototype measuring instrument for blood oxygen levels that precisely measures oxygen levels with an average error of 0.01 to 1%. It was proved that the system was efficient and helpful in monitoring patients infected with COVID-19 remotely.

The impact of using different data transmission protocols was investigated in [26]. The delays were measured in remote patient monitoring systems using Advanced Message Queuing Protocol (AMQP) and Constrained Application Protocol (CoAP) protocols. The findings indicated that the AMQP protocol can perform better for larger data packets and high-load scenarios, while the CoAP protocol was more efficient for small, frequent updates. The study underscored the need to select appropriate protocols to ensure effective and reliable data transmission in remote patient monitoring systems. With the aim of improving healthcare delivery in rural areas, the research in [27] presented an IoT-based real-time RPM system. The system used the MQTT data transmission protocol to transmit ECG data to a web server, enabling remote monitoring via smartphones or computers. The conducted experiments showed no packet loss or errors in both private (LAN) and public (WAN) networks. It supported both real-time and store-and-forward ECG monitoring, with real-time mode facing challenges like transmission delays. The use of Message Queuing Telemetry Transport (MQTT) protocol ensured low bandwidth and low power consumption.

The study [28] emphasizes the importance of reliable data delivery in the Internet of Things (IoMT) for applications like smart hospitals and traffic monitoring. It highlights the challenges of higher bandwidth and computational resources compared to traditional IoT devices, and the impact of increasing multimedia data on transmission, processing, and storage. The study also highlights the subjective quality of experience (QoE) as a significant factor in network performance evaluation, influencing perceived service reliability.

Based on the presented literature so far, it is clear that the effectiveness of IoMT-based RPM systems depends heavily on the performance of system components, mainly on the Internet service and IoT platform. Therefore, it is crucial to thoroughly test and evaluate the reliability of the specific components used in the system to ensure dependability and to recommend the system for suitable healthcare services.

3. System model

3.1 System Design: The schematic diagram of the designed healthcare IoMT system is shown in **Figure 1**. The system can be divided into three main parts. The first is the cloud platform, which serves as the system's backbone, providing communication infrastructure, data storage, retrieval, analysis, and security for all devices connected to it. The second is the healthcare room, which is designed to accommodate the patient. The medical sensors are connected to a microcontroller equipped with a Wi-Fi module. This microcontroller collects data from the medical sensors and sends it to the cloud. Another microcontroller collects environmental data and sends it to the cloud. All controllers can receive commands from a remote healthcare provider to control patient and environment-related actuators such as valves of nutrition bags, lights, and fans. Each patient microcontroller is connected to a local display screen, which displays sensor readings and emergency situation suggestions.

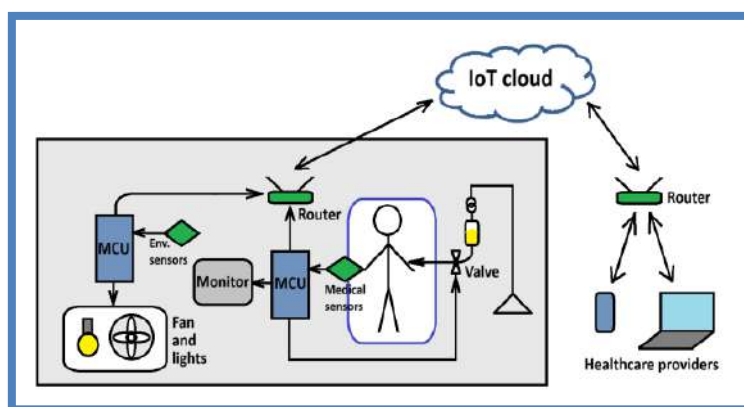


Figure 1: The designed IoMT system.

The third part of the designed system is the remote healthcare provider devices. Healthcare providers (like doctors, nurses, and home health agencies) use devices connected to the IoT

platform through the Internet to receive patient data and cloud alerts. These devices have simple, clear, and user-friendly interfaces for monitoring patients and issuing control commands.

3.2 System Workflow: The designed IoMT system uses data collection, transmission, storage, and analysis mechanisms to ensure that the data is accurate, reliable, and secure, enabling effective remote patient monitoring and healthcare delivery. Before the system is operational, the microcontrollers are set to be connected to the Internet through a local Wi-Fi wireless router. The microcontrollers are also well defined within the cloud platform, such that suitable cloud variables are assigned to the streams of collected data initiated from medical and environmental sensors. The available cloud options and services are suitably customized to correctly accommodate and display the data on the dashboards of the platform. Control buttons are added to the dashboards as a means for the healthcare providers to control actuators.

The privacy and security of data access from both sides of the designed system (sending data to the cloud and accessing dashboards for monitoring and control) are achieved by limiting access to authorized devices only according to their Media Access Control (MAC) addresses. The collected sensor data on the cloud server is analyzed, and in accordance with notifications and alerts, is generated to inform healthcare providers when abnormalities and emergency cases occur. These alerts are determined by setting upper and lower bounds for the normal ranges of every measured parameter. Alerts are initiated by the cloud server once a value exceeds normal limits. In similar systems, a monitor is also used to display the sensor readings of each patient. In this research, the function of this monitor is extended to display suggestions and recommendations in abnormal situations, and most importantly, in the case of losing Internet connectivity. These suggestions are pre-programmed inside the patient microcontroller, and they are related to the reading of the sensors according to recommendations obtained from specialized consultants.

The designed system is easily scalable. It is open to deal with more patients by just iterating replicas of the hardware and software modules used for one patient. The cloud platform can easily be modified to accommodate more patients (and maybe more sensors and functions) by upgrading the usage plan according to the available options.

3.3 System Hardware Implementation: The designed IoMT-based healthcare system is implemented using hardware and software modules to perform its specified functions. The utilized hardware components are as follows:

1. Microcontrollers: Three Node MCU esp8266 modules are used, two for patient circuits and one for environmental parameters. The characteristics of this microcontroller and

the integrated Wi-Fi module make it a suitable choice for connecting sensors and actuators.

2. Medical sensors

- The MAX30100: is a device that monitors heart rate and pulse oximetry, using light-emitting diodes and an Infra-Red sensor. It uses an analog signal processing unit to improve output signal quality and operates on input voltages from 1.8V to 3.3V. The sensor has an accuracy range of 96.17%-97.67% for blood oxygen saturation level and 92.35%-99.65% for heart rate.
- DS18B20: is a digital thermometer that offers 9-bit to 12-bit Celsius temperature measurements, with an operating temperature range of -55°C to +125°C and accuracy of $\pm 0.5^\circ\text{C}$. It connects via a one-wire bus, requiring only a single data line and Vcc and GND for microcontroller interfacing.

3. Room environment sensors

- The DHT11: is a temperature and humidity sensor with a calibrated digital output. It measures temperature in the 0°C to $+50^\circ\text{C}$ range with a $\pm 2^\circ\text{C}$ accuracy and relative humidity in the 20 to 90% range with a $\pm 5\%$ accuracy.
- MQ-135: is a gas sensor that detects harmful gases and smoke using both digital and analog output pins. The digital output gives a high signal when gas concentration exceeds a threshold, while the analog output voltage ranges from 0 to 4.2V. The detection range varies based on the gas being measured.

4. Actuators: The designed IoMT system consists of two actuators and two electronic switches. They are controlled by the healthcare providers through the control buttons on the dashboard. Two of these actuators are used to control the solenoid valves for the purpose of controlling the flow of nutritional fluid into the patients' bodies. The two switches are implemented by relays to control the ON/OFF operation of the fan and lights in the healthcare room.

4. IoMT reliability metrics

The undesired effects of communication impairments on the performance of the implemented IoMT system are the delay in data delivery, data losses and inaccuracy of received data. These effects can be measured and the reliability of the system can be assessed by evaluating several key numerical metrics. These metrics help in forming a clear idea of the extent to which the system is performing as expected. These metrics include:

- A. Latency and Response Time:** Calculating the average and maximum time required to deliver a data entity. These metrics help in understanding the range of response times and identifying any outliers or extreme delays. This metric is essential for assessing how quickly and efficiently a network operates, impacting user experience and system quality. A low reaction time indicates rapid task handling by the network. However, if

it's high, it could indicate that the network is crowded, has experienced problems, or has become ineffective [29].

B. Uptime and Availability

- Mean Time Between Failures (MTBF): This metric measures the average time the system operates without failure. It is calculated by dividing the total operational time, called the uptime, by the number of failures [30].

$$MTBF = \frac{uptime}{number\ of\ failures} \quad (1)$$

- Mean Time To Repair (MTTR): This metric measures the average time taken to repair the system after a failure. It is calculated by dividing the total time of failure, called the downtime, by the number of failures [31].

$$MTBF = \frac{downtime}{number\ of\ failures} \quad (2)$$

- Availability: Often expressed as a percentage, availability is calculated by using the formula [32].

$$Availability = \frac{MTBF}{MTBF+MTTR} \times 100\% \quad (3)$$

- C. Data Integrity:** Sometimes known as the Error Rate and it is the number of errors detected in the data transmitted or stored, usually expressed as errors per million (EPM) or simply as a percentage ratio of the number of errors to the total number sent data items [29].

5. Test procedure

In order to measure the previous metrics, the following procedure has been applied:

1. Software tools have been implemented to measure the system performance defined by delay, data loss, and EPM. The *Teraterm* serial port monitor software is used to record the values of the transmitted sensor data, together with the transmission time, to be used as a reference for the measurements. Also, as one facility offered by the cloud platform, the data on their exact arrival time is recorded.
2. The system operated for 24 hours for 10 days, and the system performance parameters were measured and recorded to observe how the system performs under various realistic network conditions. The sensor's data are replaced by previously recorded real patient data obtained from the same sensors. The data represent the sensor readings for 24 hours. The data were stored and fed to the system continuously and repeatedly throughout the period of the system test. This is done because of the difficulty in performing this test on a human for this relatively long period, and since the focus of this research is to evaluate the impact of communications impairments on system performance.
3. The collected data is systematically analyzed according to the metrics defined to assess the reliability of the system.
4. The procedure is repeated for two different cloud platforms.

6. The Results

According to the test procedure, the implemented IoMT system is first tested over the Arduino IoT platform. Artificially generated data representing the medical and environmental sensors is generated and sent to the cloud sequentially. The system is supplied by an uninterruptible power source to maintain continuous operation for 24 hours for 10 days. Internet connectivity is maintained through the popularly available optical service, with a maximum bit rate of about 12 Mbps. So, the system will operate under real conditions, and the recorded measurements can give a reasonable idea about its reliability.

The first measured parameter is the time delay, defined as the period between data transmission and reception time. The maximum delay is the performance-limiting parameter, so it is measured as shown in **Table 1**. Although in detailed tests the delay is measured for each received data item, in this table only the maximum delay within each test hour is recorded. This is because some sent data items are lost. It can be observed that there is no regular pattern. This is because the delay is the result of many interrelated parameters. The significant conclusion from this table is that the maximum encountered delay didn't exceed one minute, making the implemented system suitable for telemedical applications that allow a delay time not exceeding a minute.

Table 1: Maximum delay (sec) by using the Arduino IoT platform.

Hour	Test day									
	1	2	3	4	5	6	7	8	9	10
0	10.968	40.231	46.987	12.456	30.985	0.762	9.542	22.876	33.761	0.987
1	10.959	14.13	4.891	20.987	31.987	0.981	10.988	20.988	33.981	39.987
2	12.902	43.099	4.89	33.761	40.987	0.543	10.988	19.876	10.987	38.876
3	10.924	14.046	4.883	28.991	15.672	0.877	12.987	17.988	15.987	35.876
4	10.91	14.036	4.878	30.987	41.988	0.988	15.988	17.999	16.987	34.987
5	18.179	13.992	4.873	31.771	23.765	12.988	17.543	18.988	16.976	33.769
6	10.823	14.004	4.866	33.991	23.987	13.988	19.679	13.988	20.876	23.981
7	10.961	43.742	4.864	19.987	28.876	14.988	18.988	13.988	21.988	19.987
8	10.903	14.021	58.397	12.877	17.987	15.988	19.987	12.988	22.876	30.877
9	10.974	14.028	58.351	12.543	20.871	12.987	12.988	20.987	23.876	31.988
10	25.681	14.163	2.575	17.988	22.987	19.988	11.658	23.987	24.987	31.658
11	25.633	14.913	9.106	17.987	42.987	19.988	10.877	22.765	25.987	30.469
12	25.627	35.334	9.16	20.988	43.988	20.987	12.988	22.765	25.987	29.437
13	25.571	35.29	23.103	21.988	42.987	23.988	19.987	21.982	22.987	40.987
14	46.342	15.175	46.687	48.333	11.368	31.678	49.721	16.46	11.952	10.365
15	46.329	13.405	46.68	48.32	11.257	31.656	49.705	22.482	10.346	8.802
16	46.315	13.582	46.669	48.311	47.078	31.652	49.69	17.885	10.397	8.708
17	46.298	13.438	46.656	48.298	11.395	33.432	49.68	18.174	10.398	8.751
18	46.305	13.493	46.645	48.292	12.881	2.324	50.275	16.535	36.402	47.362
19	46.321	13.538	46.625	48.276	12.829	0.338	50.224	16.535	37.502	47.35
20	46.266	13.914	46.618	48.264	12.801	0.478	9.238	18.361	37.685	47.34
21	48.539	14.029	46.608	49.397	12.783	0.443	9.367	18.958	36.197	47.294
22	50.772	21.275	46.597	15.205	12.77	0.243	13.432	16.502	36.141	26.959
23	25.749	28.381	38.862	15.114	13.788	0.408	9.266	26.953	36.157	28.976

In order to take the variation in Internet quality of service during the day, the maximum and average time delay at each test hour (averaged over the 10 test days) are plotted in **Figure 2** versus test time ranging from 0 (12 AM) through 23 (11 PM). The general pattern shows increased delay during the times of intense use of the Internet around 9 AM and the evening period, but without exceeding 59 seconds for the whole test period. Also, the tests show an average delay between about 16 and 32 seconds over 24 hours.

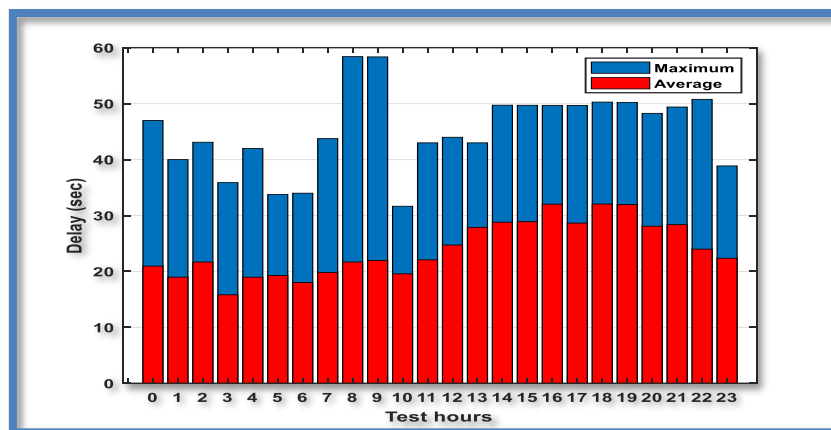


Figure 2: Delay per hour for Arduino IoT platform.

Similarly, to consider the possible daily variation in Internet service quality, the maximum and average delay within each test day, are plotted in **Figure 3**. The plot shows that the delay is not constant over different days, but instead, it is affected by the Internet service. The importance of this plot is that it gives practically encountered delay ranges, which are beneficial in evaluating and making decisions whether the implemented system is suitable for a specific telemedicine application or not.

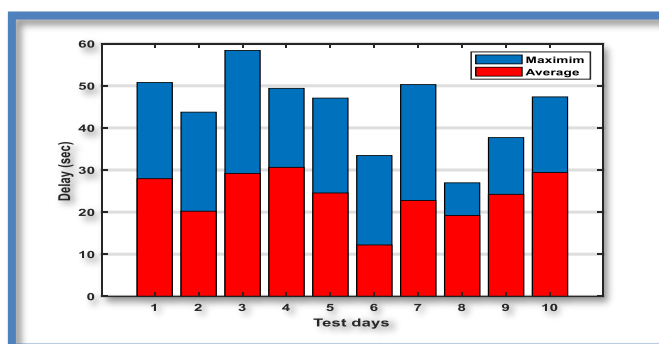


Figure 3 Delay per day for Arduino IoT platform.

Next, data loss is measured. A data item is considered to be lost if it was sent from the microcontroller and wasn't received at the platform. The percentage of the amount of lost data within each test hour is measured as shown in **Table 2**. From the first overview, it may be stated that the implemented system with the Arduino IoT platform could deliver the sensor readings with limited losses. However, odd cases of high loss percentage may practically occur, like

88.33% at hour 19/ day 7, due to unexpected interruption in Internet service, but the tests over a continuous 10 days showed that such a severe case is very rare. The data loss measurements also show the correlation with the Internet usage load. That is, relatively more data losses are expected in the morning and evening periods, which are the periods of high activity of Internet users.

Table 2: Percentage data loss for Arduino IoT platform.

Hour	Test day									
	1	2	3	4	5	6	7	8	9	10
0	0	0	1.66	1.66	0	0	0	0	0	0
1	0	0	0	0	3.33	0	0	0	0	0
2	0	0	0	1.66	0	3.33	0	0	0	0
3	0	0	0	0	0	0	0	0	0	5
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	1.66	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	15	0	0	0	0	0	0	0
9	8.33	0	13.33	0	0	0	0	0	1.66	0
10	5	0	3.33	0	0	0	1.66	0	0	0
11	0	35	1.66	0	0	0	0	0	0	0
12	0	11.66	0	0	3.33	0	0	0	1.66	0
13	1.66	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	1.66	0
17	0	0	0	0	0	5.00	0	0	0	0
18	0	0	0	0	16.66	1.66	0	0	5.00	6.66
19	0	0	0	0	0	0	88.33	0	0	0
20	0	10.00	0	0	0	0	16.66	0	0	0
21	6.66	0	0	31.66	0	5.00	0	0	0	0
22	3.33	0	3.33	6.66	0	0	0	0	0	0
23	0	0	0	18.33	1.66	0	0	3.33	0	0

In addition to the amount of data lost, it is important to take into account the time periods during which data loss fully occurs, known as the downtime, because it is one of the direct factors that determines the reliability of the system. **Table 3** shows the system reliability calculations. The uptime is the sum of the time periods during which the system is properly operational. The downtime is measured as the length of the periods during which data was being lost. Each separate downtime period is considered as a single failure, regardless of how many data items were lost within this period. For example, if the system is operating properly and suddenly a single data item (or more but continuously) is lost and then system resumed correct operation, this is considered as a single failure. On this bases, the MTBF, MTTR and hence the system availability are calculated as shown in **Table 3**. The results show that the availability ranges from 95.76% to 99.79%, with the daily average percentage data loss, calculated as the EPM, in the range from 4.44% to 0.21%. These results indicate a high level of reliability of the implemented system under realistic operation conditions.

Table 3: Reliability calculations for Arduino IoT platform.

Day	Total Up time (minutes)	Total Down time (minutes)	Number of failures	MTBF (minute)	MTTR (minute)	Availability (%)	EPM (%)
1	1425	15	5	285	3	98.96	1.04
2	1406	34	3	468.67	11.33	97.64	2.36
3	1417	23	6	236.17	3.83	98.40	1.6
4	1404	36	4	351	9	97.50	2.5
5	1425	15	4	356.25	3.75	98.96	1.04
6	1431	9	5	286.2	1.8	99.38	0.62
7	1379	61	3	459.67	20.33	95.76	4.44
8	1437	3	2	718.5	1.5	99.79	0.21
9	1434	6	4	358.5	1.5	99.58	0.42
10	1433	7	2	716.5	3.5	99.51	0.49

Moreover, in order to figure out the effect of the adopted IoT platform, the same implemented system with the same hardware and software modules has been tested over a different IoT platform, namely the well-known Blynk IoT platform. The same test procedure is applied, and the system is again tested for 24 hours for 10 days. The delay measurements are presented in **Table 4**. Fortunately, the maximum encountered delay is also less than 59 seconds, but it should be noted that on average, the system suffered from less delay time. So the implemented system over the Blynk platform seems to be relatively faster than the case of the Arduino platform. As shown in **Figure 4**, the average delay ranges from 5.5 to about 17 seconds, which is less than that in **Figure 2**. Similar comments also hold for the daily-based delay measurements shown in **Figure 5** and **Figure 3**.

Table 4: Maximum delay (sec) by using the Blynk IoT platform.

Hour	Test day									
	1	2	3	4	5	6	7	8	9	10
0	0.325	2.654	0.795	0.431	10.872	0.221	2.871	2.761	20.431	24.651
1	0.35	0.323	1.792	0.981	13.987	0.297	2.971	2.871	12.871	23.541
2	0.347	0.372	1.75	0.432	6.971	0.371	4.981	1.761	21.981	20.761
3	0.354	0.357	1.789	0.543	15.871	0.541	6.981	2.321	25.981	21.871
4	0.641	0.372	0.79	0.411	20.761	0.193	4.871	1.981	19.981	17.987
5	0.372	0.388	0.788	0.521	21.876	0.479	7.981	1.871	20.991	21.651
6	0.386	0.402	1.76	0.763	9.761	0.179	7.543	2.981	29.981	33.549
7	0.614	9.591	0.786	0.659	22.871	0.183	5.348	3.981	32.853	32.651
8	0.397	0.433	0.784	0.459	23.654	0.198	3.541	3.761	33.871	23.548
9	0.606	0.446	1.781	0.329	22.951	0.461	3.841	10.871	20.439	18.651
10	0.602	0.796	7.418	0.981	21.295	0.279	2.981	15.654	18.571	19.981
11	3.278	0.541	24.276	1.92	9.942	0.628	2.251	1.679	38.622	24.666
12	0.279	0.481	9.255	1.097	16.046	0.635	0.771	1.664	38.614	24.682
13	0.272	0.504	25.228	1.105	19.079	0.632	0.772	58.773	38.604	24.708
14	4.247	0.493	3.601	0.179	7.096	0.646	8.224	58.783	38.594	24.716
15	1.244	0.385	1.586	0.141	2.083	0.646	1.225	58.814	21.905	40.241
16	0.245	1.359	1.57	0.205	0.9399	0.683	2.212	58.808	25.09	41.23
17	1.231	0.358	1.55	0.174	2.318	0.693	2.209	4.355	25.688	24.725
18	0.931	0.34	1.532	0.199	14.698	0.69	0.805	4.345	21.566	24.737
19	0.926	0.32	1.513	0.203	1.678	0.447	0.815	3.342	21.574	24.741
20	0.92	0.3	1.496	0.219	0.668	0.47	0.823	51.491	21.574	24.788
21	28.22	0.278	1.477	0.234	0.652	0.472	0.83	26.633	21.583	24.784
22	1.771	0.259	1.456	0.248	27.922	0.465	0.994	2.632	21.608	24.788
23	1.763	0.24	1.441	0.264	11.056	0.534	1.005	2.603	21.61	24.807

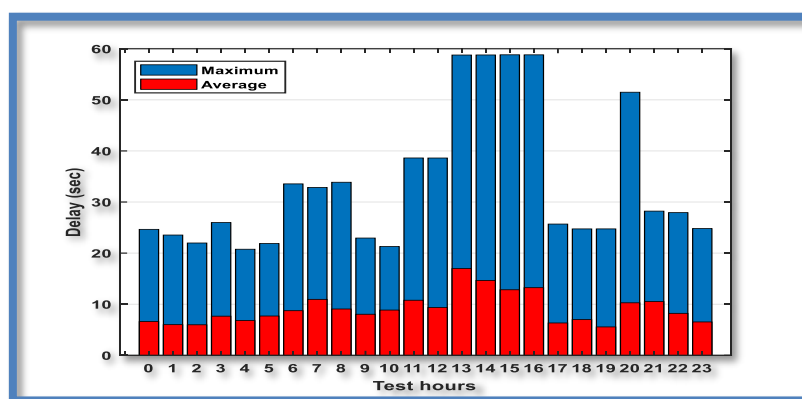


Figure 4 Delay per hour for Blynk IoT platform.

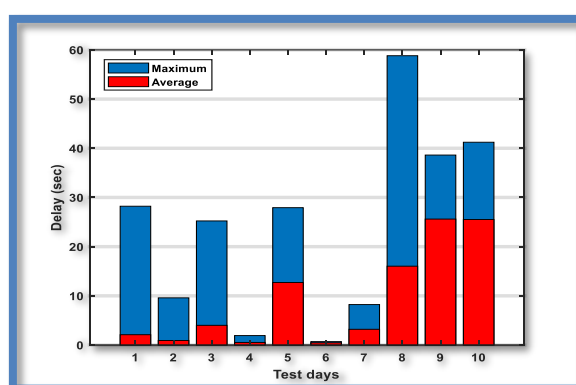


Figure 5 Delay per day for Blynk IoT platform.

Additionally, an important consideration when evaluating system latency is its comparison to commonly accepted standard latency values for various IoMT-based applications. **Table 5** outlines the key telemedicine applications and their acceptable delay thresholds. The implemented system is categorized under non-critical periodic monitoring. Based on the extensive testing, the overall average delay values not exceeding 32 and 17 seconds observed in the system fall well within the acceptable range for this class of telemedical applications.

Table 5 Latency of main telemedicine applications.

Application class	Latency requirement	Reason	Example
Critical	Low , typically less than 330 ms. [33],[34]	These applications require immediate feedback or action to ensure patient safety.	Remote surgery and urgent emergency response systems.
Non-Critical Monitoring	Moderate , typically up to several seconds. [35], [36]	These systems can tolerate delays without compromising patient safety or care.	Routine patient monitoring and wearable health devices for fitness tracking.
Long-Term Data Analysis	High , can tolerate delays of several minutes or even hours. [35]	These applications are typically not time-sensitive and are used for long-term health management rather than immediate interventions.	Predictive analytics and historical data review.

Next, the percentage data loss for the Blynk platform is measured as shown in **Table 6**. As compared with the case of Arduino IoT platform, the Blynk based system suffered from slightly more amounts of data loss. Although a maximum loss of 75% happened at hour 17/ day 10, it is clearly a rare situation related with a temporary degradation in Internet service. But, in total,

the losses in the Blynk platform are more as compared with the case of the Arduino IoT platform. This can have a negative impact on reliability.

Table 6: Percentage data loss for Blynk IoT platform.

Hour	Test day									
	1	2	3	4	5	6	7	8	9	10
0	3.33	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	5	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	1.66	0	0
5	0	0	0	0	0	1.66	0	0	0	0
6	0	0	0	1.66	0	0	0	0	0	0
7	0	0	0	0	3.33	0	0	0	0	0
8	15	0	0	0	0	0	5	0	0	0
9	13.33	41	15	0	0	0	0	0	0	1.66
10	3.33	45	16.66	0	0	0	0	0	0	0
11	0	0	1.66	0	3.33	0	0	0	0	0
12	0	0	0	0	6.66	0	0	0	0	0
13	0	0	1.66	0	3.33	0	0	20	0	0
14	0	0	0	0	0	0	0	1.66	1.66	0
15	0	0.66	0	0	13.33	0	0	0	0	1.66
16	0	0	0	0	0	0	0	20	1.66	26.66
17	0	0	0	0	10.00	0	0	0	25.00	75.00
18	10.00	0	0	0	0	3.33	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	15.00	0	0
21	3.33	0	0	3.33	0	0	0	3.33	0	0
22	0	0	0	0	1.66	0	1.66	0	0	0
23	0	0	0	0	0	0	0	0	0	0

The reliability calculations are shown in Table 7. The relatively large loss peak at the tenth test day has reduced system availability to 95.56%, which is by itself can be considered as a high percentage. For the rest of the test days, the availability is higher and reaches 99.79%. However, as a way of validation for the availability results and the related reliability metrics, the EPM is calculated as the average loss per day from Table 6. As shown in Table 7, the EPM perfectly complements the availability values.

Table 7: Reliability calculations for Blynk IoT platform.

Day	Total Up time (minutes)	Total Down time (minutes)	Number of failures	MTBF (minute)	MTTR (minute)	Availability (%)	EPM (%)
1	1411	29	7	201.57	4.14	97.99	2.01
2	1388	52	4	347	13	96.39	3.61
3	1419	21	4	354.75	5.25	98.54	1.46
4	1437	3	3	479	1	99.79	0.21
5	1415	25	11	128.64	2.27	98.26	1.74
6	1437	3	2	718.5	1.5	99.79	0.21
7	1436	4	2	718	2	99.72	0.28
8	1404	36	8	175.5	4.5	97.5	2.5
9	1420	20	4	355	5	98.61	1.39
10	1376	64	5	275.2	12.8	95.56	4.44

Based on the presented results and findings, it can be stated that the implemented system can be relied on as a successful remote patient monitoring tool built by using the available Internet service and technical components in the local region of Kirkuk city. It can positively contribute

to the healthcare sector as described in section 1. Finally, it worth noting that the specifications of the implemented system in terms of sensors types, accuracy, delay, loss rate, etc., may fit many telemedical applications whose requirements match these specifications. But at the same time, it may be not suitable for others. That is, the implemented IoMT system becomes not suitable for telemedical applications that are delay sensitive and do not have the suitability to withstand such values of time delay, data loss, etc.

7. Conclusions

In this paper, an IoMT system has been designed and implemented for remote patient health monitoring. The system has been tested to evaluate its reliability under practical conditions. Two different popular IoT platforms, namely the Arduino and Blynk, have been used to compare their impact on the performance of the system. The conducted tests have provided a practical experimentation results for the implemented system under realistic operation conditions and locally available facilities. Data is sent in this system with a variable time delay depending on the connection status, but it does not exceed 59 seconds in the worst circumstances. The tests show the Blynk IoT platform cause less delay as compared with the Arduino IoT platform. Also data transmission involved losses, and although rarely, high loss rates may appear. But the tests for 24 hours over 10 days have shown that the system can achieve average availability of more than 98% for both of the tested platforms. Therefore, as a conclusion, it can be stated that the tested platforms seem to be equivalently reliable, with Blynk having an advantage in terms of the delayed time.

However, the implemented system is not without limitations. On the shelf sensors were used for two reasons: first to prove the concept with a limited budget, second their accuracy is suitable for the target RPM system. Even though, the used sensors and circuitry have enabled a high reliability system.

Possible future work may be applied to test different system configurations to suit different telemedical applications, such as tele-surgery, requiring more accurate sensory, less delay and less loss rate. However, performance improvement by examining selective system customizations could be a wide field of many future works. The cost of the system may be studied and analyzed to optimize system resources according to different requirements and practical limitations. Further system performance improvement may be investigated by examining the individual and collective effects of specific factors and system components.

In summary, the implemented system has been built by using locally available resources. The tests showed it is accurate enough and reliable for RPM with limited delay and loss rate. It is customizable to suit a wide range of medical applications. Therefore, a possible collaboration

with the healthcare agencies in Kirkuk city may be held in order to bring the system into actual practical operation to support and improve the healthcare system.

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Landfill Site Suitability Assessment Based on GIS and Multicriteria Analysis: A Case Study of Kirkuk City

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Keywords: Landfill, Geographic Information System (GIS), Analytic Hierarchy Process (AHP), Convolutional Neural Network (CNN), Kirkuk, Multicriteria Analysis, Suitability Assessment.

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Abstract:

This study looks at the environmental and socioeconomic aspects of possible landfill locations in Kirkuk City, Iraq, as well as their spatiotemporal appropriateness. This study used different types of data, including Landsat satellite imagery, soil texture, groundwater level, and slope. The Analytic Hierarchy Process (AHP) was utilized for multi-criteria decision analysis of possible landfill sites, linear regression was employed for population projection, and a Convolutional Neural Network (CNN) was utilized for Normalized Difference Vegetation Index (NDVI)/ Normalized Difference Built-up Index (NDBI) prediction. The suitability ratings for prospective dump sites were produced using the AHP-based Geographic Information Index (GIS) techniques. The results reveal that the selection of landfill locations minimizes environmental effects and advances environmentally sound waste management. The technique provides a framework for assessing the appropriateness of dump sites in various geographical areas. Moreover, the projections for the future emphasize Kirkuk City's need for upgraded waste management facilities. Furthermore, urban planners and politicians in Kirkuk City may benefit greatly from this research's data-driven approach to landfill site selection, which takes social and environmental concerns into account and has implications for sustainable waste management techniques.

Keywords: Landfill, Geographic Information System (GIS), Analytic Hierarchy Process (AHP), Convolutional Neural Network (CNN), Kirkuk, Multicriteria Analysis, Suitability Assessment.

تقييم مدى ملائمة موقع المكب بناء على نظم المعلومات الجغرافية وتحليل متعدد المعايير: دراسة حالة مدينة كركوك

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الخلاصة:

نظرت هذه الدراسة في الجوانب البيئية والاجتماعية والاقتصادية لمواقع مدافن النفايات المحتملة في مدينة كركوك بالعراق، فضلاً عن ملائمتها الزمانية المكانية. تم استخدام أنواع مختلفة في هذه الدراسة بما في ذلك صور الأقمار الصناعية لاندسات، وملمس التربة، ومستوى المياه الجوفية، والمنحدر. واستخدمت عملية التسلسل الهرمي التحليلي (AHP) لتحليل القرار متعدد المعايير لمواقع المكب المحتملة، واستخدمت الانحدار الخطي لإسقاط السكان، واستخدمت الشبكة العصبية التلافيفية (CNN) للتنبؤ ندفي/ندبي. وأنتجت تقييمات الملاءمة لمواقع التفرغ المرتقبة باستخدام تقنيات نظم المعلومات الجغرافية القائمة على برنامج التكيف الهيكلي. تم الكشف عن النتائج أن اختيار مواقع مدافن النفايات يقلل من التأثير البيئي ويعزز الإدارة السليمة بيئياً للنفايات. وتوفر هذه التقنية إطاراً لتقييم مدى ملائمة مواقع تفرغ النفايات في مختلف المناطق الجغرافية. علاوة على ذلك، تؤكد التوقعات المستقبلية على حاجة مدينة كركوك إلى تحسين مرافق إدارة النفايات. علاوة على ذلك، قد يستفيد المخططون والسياسيون الحضريون في مدينة كركوك بشكل كبير من نهج هذا البحث القائم على البيانات لاختيار مواقع دفن النفايات، والذي يأخذ في الاعتبار الاهتمامات الاجتماعية والبيئية وله آثار على تقنيات الإدارة المستدامة للنفايات.

الكلمات المفتاحية: مكب النفايات، نظم المعلومات الجغرافية، التسلسل الهرمي التحليلي، الشبكة العصبية التلافيفية، كركوك.

1. Introduction:

Every nation on the planet deals with a variety of environmental issues [1]. Municipal solid waste management is becoming a global concern that many municipalities are facing, especially in hilly cities. Additionally, the UN's goal for global sustainability now includes municipal waste management as an emerging problem because of the population increase [2–4].

The aim of solid waste management is to dispose of waste in the most environmentally friendly way feasible [5]. The locals who are immediately touched by a region's solid waste program help to achieve this. Households, workplaces, small businesses, and commercial establishments all have their solid trash collected. This is regarded as a unique waste stream in the EU. The term "MSW" refers to this type of trash as well as waste produced during building, restoration, and demolition. MSW includes items like glass, bricks, concrete, inert materials, paper and cardboard, yard trash, metal, plastic and rubber, electronic waste, and miscellaneous garbage. Municipalities across the world classify MSW in different ways. It is composed of both

biodegradable and non-biodegradable elements as well as organic and inorganic components. Many different approaches are used worldwide to reduce the production of solid waste. The most popular strategies for reducing solid waste are garbage disposal, recovery, reuse, recycling, and prevention. Another tactic used to prevent any environmental problems is the regular storage of solid waste. One of the major worldwide concerns that is particularly noticeable in developing countries is the disposal of municipal solid waste (MSW) [6]. Finding, evaluating, and planning for landfill sites is one of the most important steps in the MSW disposal process [7,8].

According to their research, the number of households, employment, and population expansion all have a big influence on trash generation rates. Predicting a 70.6% rise in garbage by 2031, the LSTM model proved to be the most accurate in predicting future trends in waste. Planning effective waste management systems to accommodate the predicted rise in trash creation would need careful consideration of this forecast.

The aim of this study is to provide a framework for assessing the appropriateness of dump sites in various geographical areas. This framework is based on the Analytic Hierarchy Process (AHP) [9], which was utilized for multi-criteria decision analysis of possible landfill sites, linear regression was employed for population projection, and a convolutional neural network (CNN) [10] was utilized for NDVI/NDBI prediction [11–13].

2. Literature Review

A number of review papers discussed GIS-based multicriteria spatial decision support systems for landfill siting suitability analysis. The factors considered for utilizing MCDA for landfill site suitability assessment [16]. The use of MCDA in tackling waste management problems emphasized the increasing complexity of waste management due to growing waste volumes and environmental concerns [17]. The review covered various MCDA techniques used in real-life waste management scenarios, highlighting their advantages and disadvantages compared to other approaches .

In [18] it was discussed that the challenges in solid waste management due to the increasing quantities and complexity of waste generated worldwide. They presented the importance of MCDA models in addressing the various dimensions and conflicting criteria involved in waste management. The study reviewed the current practices and methods of MCDA in SWM, emphasizing the need for tools to assess system performance comprehensively [18].

Abujayyab et al. (2016) discussed GIS modelling for landfill site selection, focusing on the criteria and methods used for selection as the key challenges in determining landfill-siting input criteria, as discussed in the article, include the complexity of procedures, the need to consider

environmental, economic, and social impacts, compliance with regulations, public acceptance, and the extensive evaluation required for selecting suitable landfill sites [19].

Abujayyab et al. (2017) explored the use of MCDA in GIS modeling for landfill site selection from 1997 to 2014 as the study focused on stages such as the selection of weights and decision rules in the models [20]. It identified strengths and limitations of using MCDA for landfill site selection and suggested the potential use of Artificial Neural Networks (ANN) for enhanced validation and accuracy [20].

Mat et al. (2017) discussed the criteria and decision-making techniques used in solving landfill site selection problems, it highlighted the importance of selecting appropriate landfill sites and the use of GIS and MCDA techniques in this process [21]. More recently, the review by Mohammed et al. (2019) discussed how the combination of GIS-based tools and decision analysis techniques has significantly expanded in various fields of research over the last few decades, allowing for the automation and analysis of spatial data [22].

3. Materials and methods:

3.1 Overview: The process for determining whether possible landfill sites are suitable and analyzing the spatiotemporal dynamics of several elements' influence is shown in **Figure 1**. The technique incorporates many data sources, such as population data from 2004 to 2024, Landsat satellite photos from 2004, 2014, and 2024, and climatic data including air temperature, rainfall, relative humidity, wind speed, and wind direction. Furthermore, soil texture, steady groundwater level, and slope GIS data are used.

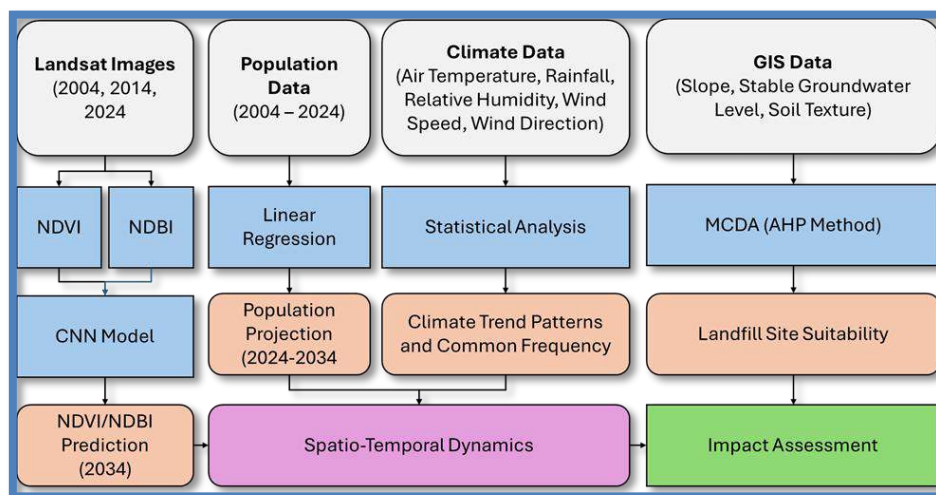


Figure 1: Flowchart of the proposed methodology overview

3.2 Study Area: This research focuses on Kirkuk City, the capital of the Kirkuk Governorate, which is situated in northern Iraq, along with its surrounding neighborhoods, including Taza and Leylan. Situated between the flat plains and the foothills of the Zagros

Mountains, Kirkuk City is located at 35°28'N 44°25'E. With a population that is expected to surpass 1.1 million by 2023 due to both natural growth and migration, this large metropolitan area is experiencing tremendous population increase. Significant urban sprawl has occurred in the city and its environs, with industrial zones primarily located on the periphery and residential and commercial sectors predominating in terms of land use. Figure 2 shows the study area.

Geographically, Kirkuk City is bordered to the north by the Zagros Mountains, to the east by the city of Sulaymaniyah, to the south by the Hamrin Mountains, and to the west by the Lower Zab Mountains. The area is around 250 kilometers away from Baghdad. The three districts of Kirkuk City, Laylan, and Taza comprise the 1368.34 km² that make up the Kirkuk study area. Its landscape is varied, with heights ranging from 192 to 713 meters above sea level and features both flat sections and quite steep hills.

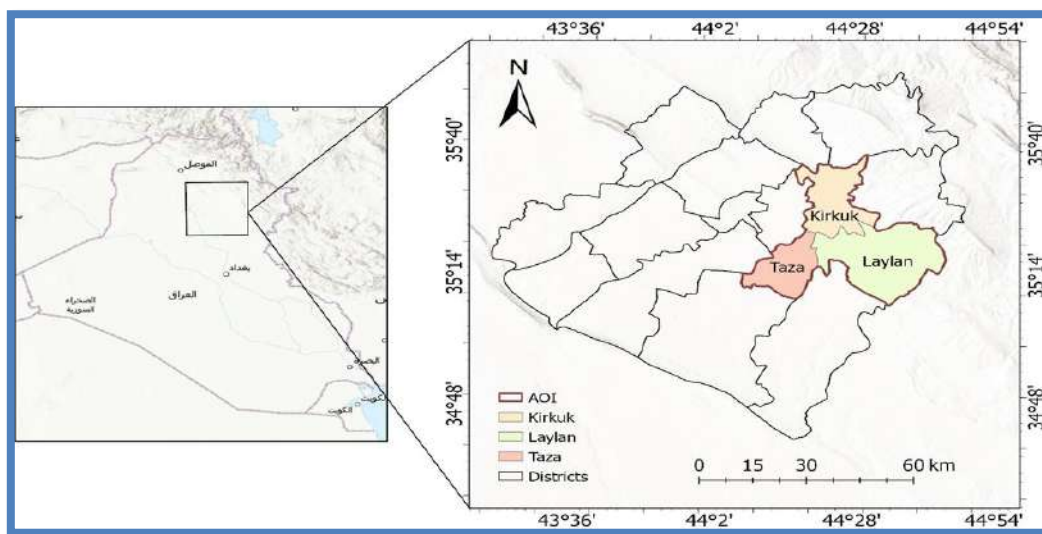


Figure 2: Map of the study including three districts of Kirkuk province (Kirkuk city, Laylan, and Taza).

3.3 Datasets

3.3.1 Landsat Data: Four images total from Landsat satellites were used in the investigation. As shown in **Table 1**. The dataset contains atmospherically corrected surface reflectance and land surface temperature derived from the data produced by the Landsat 8 OLI/TIRS sensors.

Table 1: Information on Landsat images obtained for this study.

Landsat Image	Sensor	Row	Path	Date	Cloud Cover	Processing Level
Image 2004-1	LANDSAT 5 TM	035	169	2004-03-18	18.00	L1TP
Image 2004-2	LANDSAT 5 TM	036	169	2004-03-18	2.00	L1TP
Image 2014	LANDSAT 8 OLI TIRS	035	169	2014-03-28	30.88	L1TP
Image 2024	LANDSAT 8 OLI TIRS	035	169	2024-03-01	1.35	L1TP

3.3.2 Population Data: The Department of Urban Planning in Kirkuk provides the demographic figures. The information was collected annually between 2004 and 2023. In terms of population, Kirkuk is the largest of the three cities. Between 2004 and 2023, the population

grew gradually, from 566,000 to 1,075,000. According to the data, the population is growing by between 30,000 and 40,000 people every year on average. Although Taza is a smaller city than Kirkuk, its population has been increasing more quickly. The population of Taza rose from 12,715 in 2004 to 51,119 in 2023. With the population growing by about 2,000 per year in the earlier years and then by about 3,000 to 4,000 per year in the more recent years, the growth rate seems to be quickening. In terms of population, Laylan is the smallest of the three cities. Between 2004 and 2023, its population grew from 14,887 to 20,881. The population of Laylan is growing by a few hundred each year, which seems to be a slower pace of growth than that of Kirkuk and Taza.

3.3.3 Climate Data: The Kirkuk station's meteorological section provides climatic data. The monthly statistics were collected between 2014 and 2023. Included in the climatic variables are air temperature, precipitation, relative humidity, wind direction, and wind speed.

3.3.4 GIS Data: Three primary factors—the soil texture and steady groundwater level—were taken into consideration while determining the viability of a landfill site. The research area's DEM data was used to derive the slope. With a spatial resolution of 30 m, the DEM was acquired from ASTER GDEM (<https://gdemdl.aster.jspacesystems.or.jp/>). The soil texture was taken from a 1:250000 scale raster map of the Iraqi Geological Survey.

3.4 Methods

3.4.1 Remote Sensing Indices: This investigation made use of several indices. These consist of NDBI and NDVI. The density of greenness on the ground surface is indicated by the NDVI measure, which is why it was chosen in this study. As such, it is an important factor to take into account while choosing a suitable landfill site. Equation (1) was utilized to create the NDVI using the Landsat-8 OLI dataset, as shown in reference [13].

$$NDVI = \rho_{\text{green}} - \rho_{\text{NIR}} / \rho_{\text{green}} + \rho_{\text{NIR}} \quad (1)$$

Where ρ_{green} and ρ_{NIR} are the green and the near infrared satellite image bands, respectively.

The size and geographic distribution of the research area's urban built-up areas are evaluated using the NDBI and related computations. It also offers a thorough overview of the land cover in cities. This study's NDBI map was produced using the Landsat-8 OLI dataset. For relevant computations, the near-infrared (NIR) and shortwave infrared (SWIR) bands were used, as shown in Equation (1) [14].

3.4.3 CNN

3.4.3.1 CNN Background: Convolutional operations are used on 2D input data, such as photographs or spatial grids, in order to extract significant features and patterns. This is the fundamental theory of CNNs [15]. In a regular Neural Network, there are three types of layers:

Input Layers: It's the layer in which we give input to our model. The number of neurons in this layer is equal to the total number of features in our data (number of pixels in the case of an image). **Hidden Layer:** The input from the Input layer is then fed into the hidden layer. There can be many hidden layers depending on our model and data size. Each hidden layer can have different numbers of neurons, which are generally greater than the number of features. The output from each layer is computed by matrix multiplication of the output of the previous layer with learnable weights of that layer and then by the addition of learnable biases, followed by an activation function, which makes the network nonlinear. **Output Layer:** The output from the hidden layer is then fed into a logistic function, like sigmoid or softmax, which converts the output of each class into the probability score of each class.

3.4.3.2 Patch Extraction: The input images are usually processed in discrete areas, known as patches, using CNNs. The input image is divided into smaller patches, or tiles, as part of the patch extraction process. These patches are then fed into the CNN's convolutional layers. The input picture is frequently padded with zeros around the borders prior to patch extraction. The purpose of this padding is to guarantee that patches close to the image's edge may be removed without running out of data. The size of the patches being extracted determines how much padding is needed.

3.4.3.3 Proposed CNN Architecture: One convolutional layer, one flattening layer, and two fully connected dense layers make up the CNN design that the study suggests. With data from 2014 and 2024, this CNN model is intended to forecast the NDVI and NDBI for the year 2034. The input data is passed through 32 convolutional filters of size 2x2 by the model's first layer, a 2D convolutional layer. The input shape is (3, 3, 2), where the number of channels (corresponding to NDVI and NDBI data) is represented by 2 and the spatial dimensions of the input data are represented by 3x3. This layer's output is subjected to the ReLU activation function. The convolutional layer's output is flattened into a 1D vector by the flattening layer, which prepares it for the fully connected layers that follow. The ReLU activation function is applied to the output by the following fully connected layer in the model, which consists of 16 neurons.

3.5 Validation: The actual observed data for 2024 was compared with the model's predictions as part of the validation procedure. The NDVI and NDBI values for 2024 were predicted by the algorithm after it had been trained using data between 2004 and 2014. The performance of the model was then assessed by comparing the projected values for 2024 with the actual observed values for 2024.

4. Results and Discussions

4.1 Results of NDVI/NDBI Prediction: Figure 1 shows the NDVI changes for a study region in 2004, 2014, and 2024. Higher values (darker green) in the NDVI indicate denser vegetation, whereas lower values (yellow/brown) indicate scant or no vegetation, such as in urban or desolate environments. A variety of plant densities were observed in the study region in 2004. Some areas, especially in the northern and southern parts, have extremely thick vegetation (dark green). Significant regions with moderate to low plant cover, which are represented by yellow and lighter green hues, are found as well; these areas are thought to be urban or developed areas. The research area's total vegetation density seems to have decreased by 2014. Particularly in the central and eastern sections, the forested areas have given way to urban or arid areas. The size of the dark green, high NDVI zones has shrunk, suggesting more urban or arid territory. The trend of declining vegetation density is expected to continue in 2024. A reduced percentage of the study area is currently covered by the highest, darkest green NDVI values, especially in the central and eastern parts. Comparing this year to last, there has been greater growth in the yellow, arid/urban regions. The NDVI variations between 2004 and 2024 indicate that the study area's vegetation is gradually getting less dense and dispersed. This may be brought on by things like changes in farming techniques that result in less plant cover, deforestation, or urban growth.

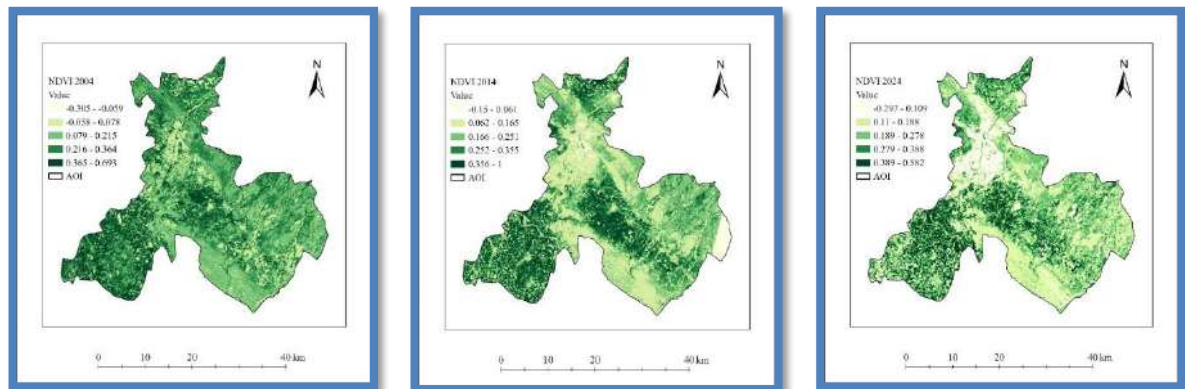


Figure 3: NDVI maps of the study area, (a) 2004, (b) 2014, and (c) 2024.

The maps in Figure 1 depict the NDBI for the research area for the years 2004, 2014, and 2024. Oranges and reds denote greater values, indicating the presence of urban and arid areas. The NDBI values are represented using a color scale. Over the course of the three years, the mapped area's two main regions, the center and the southeast, have seen the greatest concentration of urban and desolate lands, shown by orange and red. These areas probably belong to large cities or metropolitan areas. The greater regions covered in orange and red on the 2014 map compared to the 2004 image indicate an increase in the size of urban and desolate

lands between 2004 and 2014. The growth of urban and arid regions is evident in the 2024 image, which has even more areas covered in orange and red than the 2014 map had. The temporal variations point to a continuous pattern of urban sprawl and the spread of arid areas over time, with the pace of expansion seeming to be faster between 2014 and 2024 than it was between 2004 and 2014.

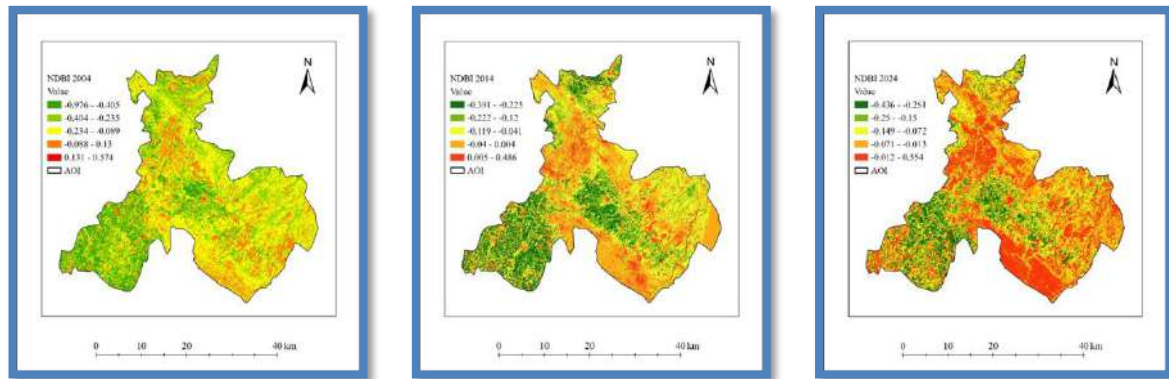


Figure 4: NDVI maps of the study area, (a) 2004, (b) 2014, and (c) 2024.

The NDVI for the research region in 2024 and the expected NDVI for 2034 are displayed in Figure 3. The NDVI readings show different degrees of plant density and health, ranging from -0.297 to 1 (green to red hues). Lower values (redder colors) denote scant or stressed vegetation, whereas higher values (greener shades) indicate denser and healthier plant cover. In comparison to 2024, the regional distribution of vegetation appears to have shifted dramatically, with some places seeing a loss of vegetation and others seeing a gain. The yellow/beige tint indicates where certain regions with moderate to high NDVI values in 2024 are expected to have little to no vegetation in 2034. The 2034 NDVI forecast shows that there won't be any appreciable changes to the plant cover.

These plant cover changes may have an influence on landfill siting because of the following possible effects. Because of possible ecological effects, places with sparse vegetation can be a better fit for landfill development than those with extensive vegetation. The anticipated alterations in the plant cover can be a result of infrastructure growth, urbanization, or other changes in land use that could affect whether possible landfill locations are suitable. To reduce soil erosion and regulate surface runoff, vegetation cover is essential. Variations in plant types may have an impact on surface water runoff and leachate management at landfills. During the site selection process, it may be necessary to take vegetation into account since it might affect the visual effect and public impression of a possible landfill site. Significant changes in the amount of vegetation might have an influence on the surrounding ecosystems, sometimes necessitating mitigation strategies or careful site selection to reduce effects on species or habitats that are vulnerable.

The NDBI for 2024 and the expected NDBI for 2034 are depicted in Figure 4. Warmer hues (yellow, orange, and red) indicate higher amounts of built-up or barren regions, whereas cooler colors (green) indicate vegetated areas. The NDBI values vary from -0.436 to 0.1. Compared to 2024, there seems to have been a considerable shift in the geographic distribution of developed and undeveloped regions, with certain places becoming more urbanized. It is anticipated that certain places with moderate to low NDBI values in 2024 will have high NDBI values (red) in 2034, suggesting a rise in built-up or bare areas, most likely because of urban growth or expansion. It is anticipated that certain regions with high 2024 NDBI values would have lower 2034 NDBI values (green), indicating a possible reduction in built-up areas or opportunity for revegetation or redevelopment.

The alterations in developed and undeveloped regions may have consequences for the location of landfills because of the subsequent elements and their effects. While locations with less built-up areas may provide more possible sites, areas with rising urbanization may have limited acreage available for landfill construction. Transportation networks and infrastructure may be easier to reach in urbanized regions, which might make it easier to move garbage to possible disposal locations. Higher densities of impermeable surfaces may be found in highly developed regions, which might increase surface runoff and require more thorough stormwater management at landfill sites. Potential landfill locations may get closer to residential areas because of increased urbanization, which might cause residents to worry about noise, odor, and other annoyances. Redevelopment of brownfields and the cleanup of previously constructed sites might be opportunities in regions with declining built-up areas and should be taken into consideration when choosing where to locate landfills.

4.2 Results of Landfill Site Suitability Assessment: A weighted average of the soil type weights, stable groundwater level, and criteria slope from earlier studies is used to assess the area's appropriateness for landfills. A map illustrating the research region's landfill suitability is shown in Figure 5. The appropriateness is broken down into five categories, as indicated by the map: very low, low, moderate, high, and very high. Several colored zones on the map represent different degrees of appropriateness for the establishment of a landfill. The areas for the five appropriateness levels—219.25, 305.30, 181.49, 304.10, and 354.84 km²—are displayed in the chart. The research region's northern regions exhibit varying degrees of eligibility for disposal locations. The red and yellow hues indicate that the region's eastern and southeast have very low and low suitability ratings, respectively. The land's southernmost portion is more favorable—in fact, exceptionally suited. Four landfill sites were chosen based on the suitability map and the accessible lands (**Figure 5**).

Site 3 is the ideal option since it implies typically flat terrain with low population density and a shallow water table, among other critical features. It has the largest population (16.922), the lowest slope (1.955), the highest landfill compatibility score (0.316), and a stable groundwater level (204.159m). It is also the closest to cities (3522.0m) and water (817.6m), which may or may not be useful depending on the circumstances. Even yet, the present landfill is rather far away (15120.5m). Site 1 is feasible as well, having the lowest slope (1.867), population (9.909), and NDVI (0.101, indicating little vegetation), in addition to having an intermediate landfill suitability (0.283). Although it is closer to water (1192.2m) than Sites 2 and 4, its closeness to towns and cities (2878.6m and 3735.4m) may provide issues.

Table 2: Weights for the landfill site suitability criteria provided by the experts questioned in this study and AHP methodology.

Main Criteria	Weight	Sub-criteria	Sub-Weights
Slope	0.310	0 - 3	0.163
		3 - 6	0.111
		6 - 9	0.034
		9 - 35.638	.
Groundwater stable level	0.333	0.388 - 112	0.173
		113 - 224	0.136
		225 - 336	0.018
		337 - 448	0.005
		449 - 560	.
Soil types	0.356	Iniana Formation	0.050
		Kirkuk group	0.011
		Valley Fill Denosits	0.038
		Sheet Runoff Denosits	0.072
		Bai Hassan Formation	0.026
		Slope Denosits	0.074
		Polv Genetic	0.072
		Jaddala and Avanah Formation	0.011
Sum	1		1

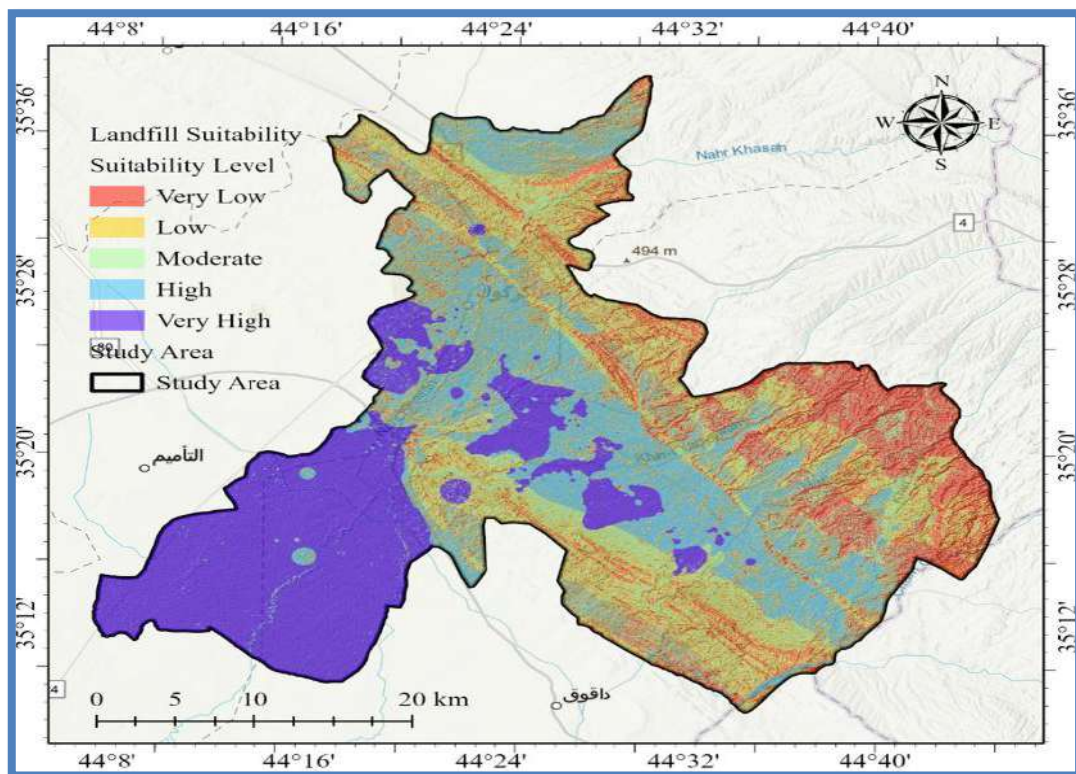


Figure 5: Map of landfill suitability in the study area.

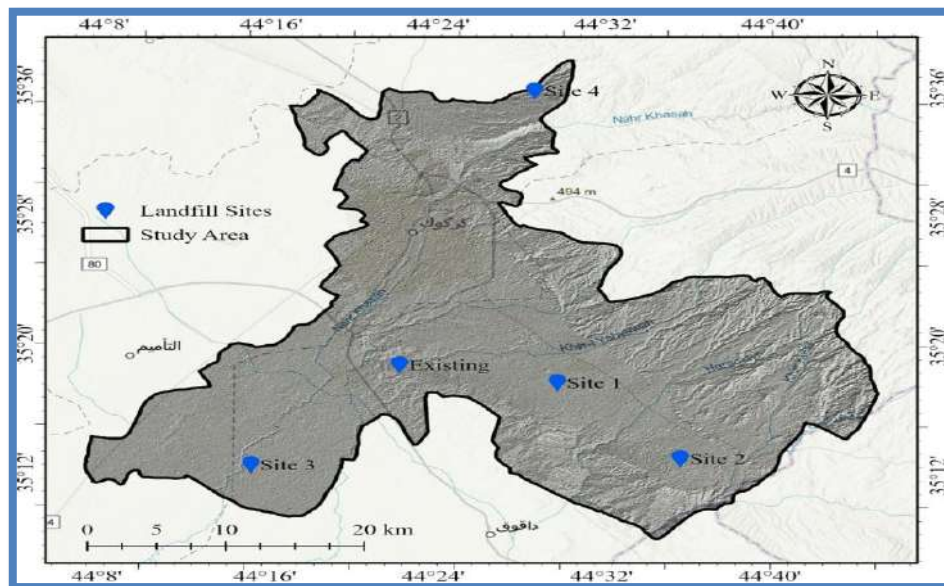


Figure 6: Map of landfill site locations including the existing and selected sites.

4. Conclusions

Significant trends in land use changes, climatic patterns, and population increase were found in Kirkuk City throughout our analysis. These patterns point to a significant rise in solid waste creation in the future, as do population estimates. The study developed a methodology for the spatiotemporal assessment of possible landfill sites to overcome this difficulty. This approach considers social, economic, and environmental aspects, such as land cost, population density, soil texture and groundwater levels. The study selected landfill locations that minimize environmental effects and enhance sustainable waste management by combining these through a multi-criteria decision analysis. For Kirkuk City urban planners, this study offers insightful information. The methodology of data-driven landfill site selection provides a model for comparable assessments in other areas. Moreover, the anticipated rise in trash production highlights the pressing necessity for enhanced waste management facilities inside the city. By putting our suggestions into effect, Kirkuk City's waste management procedures may have a more sustainable future.

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Assessing The Environmental Quality of Kirkuk City and Taza District Based on Pressure-State-Response Framework for Winter 2023 Using Remote Sensing and GIS

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Abstract:

Evaluating a region's Ecological Environment Quality (EEQ) is an essential factor in deciding its urbanization and sustainable development rate. This study aims to find the Ecological Index (EI). It evaluates it using the widely used Pressure-State-Response (PSR) framework based on a set of statistical and remote sensing indices in Kirkuk City and Taza district. Sentinel-2 satellite images were used to obtain 12 indicators that offer a foundation for sustainable development decision-making for Kirkuk City and Taza District during the winter of 2023. The finding reveals that the ecological condition is healthy in winter due to the atmospheric conditions and the social and economic activities. It presents the main relation between environmental health and human activities.

Keywords: Eco-Environment, Ecological Index, PSR Framework, Response Indicator, Sentinel-2.

تقييم جودة النظام البيئي في محافظة كركوك و مقاطعة تازة بالاعتماد على طريقة *Pressure-State-Response framework* لشتاء ٢٠٢٣ باستخدام التحسس النائي و نظم المعلومات الجغرافية

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قسم تقنيات هندسة المساحة\الكلية التقنية الهندسية-كركوك\ الجامعة التقنية الشمالية/ العراق

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الخلاصة:

تقييم النظام البيئي لمنطقة ما، هو عامل مهم في تحديد معدل التطور العمراني والتنمية المستدامة. تهدف هذه الدراسة لإيجاد المؤشر البيئي (EI) وتقييمه بواسطة الطريقة الشائعة *pressure-state-response (PSR) framework* بالاعتماد على مجموعة من المؤشرات الإحصائية والخاصة بالتحسس النائي في مدينة كركوك ومقاطعة تازة. تم استخدام الصور الفضائية *sentinel-2* للحصول على ١٢ من المؤشرات التي تم صنعها من التحسس النائي لتقييم جودة النظام البيئي. هذه المؤشرات توفر أساساً لصناعة القرار في التنمية المستدامة كتقنية حديثة تدعم التغير الذي يحدث على المدى البعيد من خلال انشاء الخرائط وعرضها تقوم هذه الدراسة بعرض المؤشر البيئي لفصل الشتاء لسنة ٢٠٢٣ في محافظة كركوك ومقاطعة تازة. يشير البحث الى ان الحالة البيئية تبدو أكثر صحة في فصل الشتاء وذلك تبعاً للظروف الجوية والنشاط الاجتماعي والاقتصادي للإنسان بالإضافة الى ان البحث يظهر العلاقة الرئيسية بين الصحة البيئية والنشاط البشري في مختلف الظروف الجوية.

الكلمات المفتاحية: البيئة، المؤشر البيئي، طريقة *PSR*، مؤشر الاستجابة، صور سينتينال-٢.

1. Introduction:

Over the past fifty years, the Ecological Index has become a crucial policy tool for efficient environmental management and pollution control [1]. The region's eco-environmental quality is crucial for sustainable socioeconomic development, ensuring harmony between social production and living environment, and assessing its ability to sustain long-term growth [2]. The natural environment is crucial for the survival and progress of humanity, as it provides vital resources such as water, land, biological resources, and climate [3]. Remote sensing technology is crucial for ecological monitoring in developing countries due to its comprehensive and dynamic compliance with environmental changes [4]. On the other hand, The PSR model highlights the impact of human activities on ecosystem health, including social pressures and resource depletion, highlighting the significant role of human activities [5]. An ecosystem's health status is determined by its vitality, structure, and resilience, while response indicators show how it reacts to changes caused by human activities and the ecosystem itself [6]. Factors like soil temperature, land use, vegetation, heat, soil texture, and aridity influence plant growth, impacting the entire ecosystem if disturbances occur [7]. Changes in land use, such as residential

expansion, can lead to increased environmental conditions. However, manmade and natural factors, such as population growth and economic development, also influence these variations [8]. Land use and cover maps are vital for improving our comprehension of environmental modeling and water management [9]. Theories on health, safety, well-being, residential contentment, and urban environment are derived from historical similar researches and decision-making in urban areas [10]. This study evaluates the eco-environmental system in Kirkuk City and Taza District, focusing on its impact on the impact of human activities and weather condition on the ecological health. It compares five ecological responses in winter 2023, preparing for serious measures.

2. Materials and methods:

2.1. Study area: Figure 1 shows Kirkuk city and Taza District which are situated in the northwestern region of Iraq. It is bordered by the Zagros Mountains to the north, the Hamrin Mountains to the south, the Lower Zab Mountains to the west, and Al-Sulaymaniyah City to the east. Kirkuk City is located at a latitude between $35^{\circ}13'$ and $36^{\circ}29'$ N and a longitude between $44^{\circ}00'$ and $44^{\circ}50'$ E. The city has a total size of around 9,679 km² [11]. The research area experiences a semiarid and Mediterranean climate with hot summers and cold winters, with a heavy precipitation peak from December to March [9]. Temperature plays a significant role in the climate which drops to a low of -1°C in the winter. The city of Kirkuk is situated in the hilly northern part of the Kirkuk plain, approximately 340-360 meters above sea level.

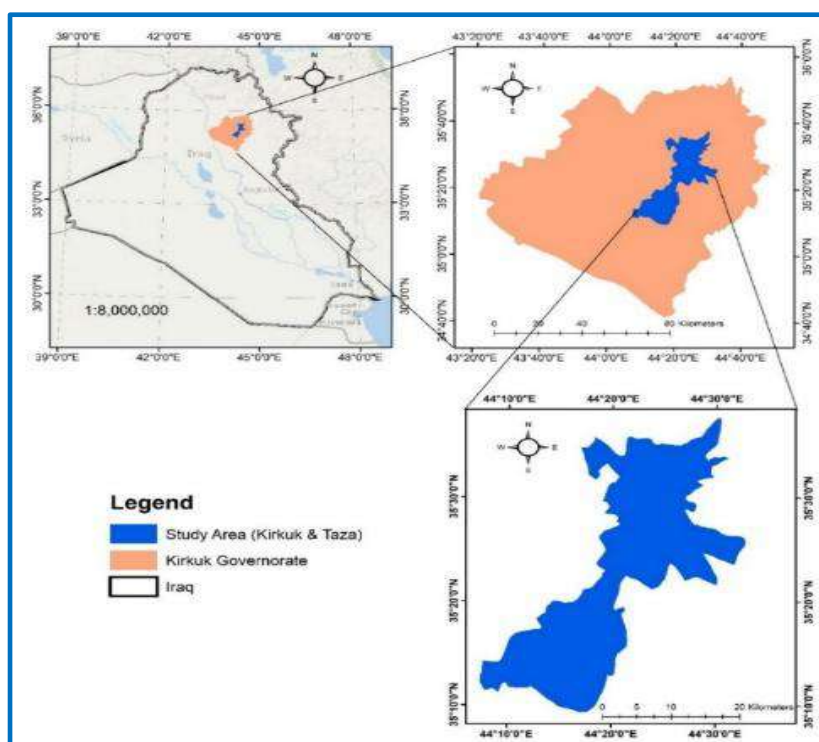


Figure 1: The Study Area of Kirkuk City and Taza District.

2.2. Data and pre-processing

2.2.1. Data used: The study utilized Sentinel-2 data, specifically level-2A images, from a multispectral sensor with 13 channels, ranging from 10 to 60 meters (image details in [Table 1](#)). All images were downloaded for free from the Copernicus website (a part of the European Union's space program) with a cloud coverage of less than 2%.

Table 1: Band details of sentinel two satellite image

Bands (wavelength region)	Central wavelength (nm)	Resolution (m)
Band-1 (coastal aerosol)	443	60
Band-2 (blue)	490	10
Band-3 (green)	560	10
Band-4 (red)	665	10
Band-5 (vegetation red edge)	705	20
Band-6 (vegetation red edge)	740	20
Band-7 (vegetation red edge)	783	20
Band-8 (NIR)	842	10
Band-8A (vegetation red edge)	865	20
Band-9 (water vapour)	945	60
Band-10 (SWIR-Cirrus)	1375	60
Band-11 (SWIR)	1610	60
Band-12 (SWIR)	2190	20

2.2.2. Data Pre-processing: The Bilinear interpolation method was used to resample photos, enhancing their quality and visual appeal. Geometric and atmospheric corrections were not made, as the images are already corrected for surface reflectance. The ortho-images, or granules, have dimensions of 110x110 square kilometers and are projected using the UTM/WGS84 coordinate system as well as registered.

2.3.Methods: The method adopted in this study to obtain the ecological index uses the pressure-state-response (PSR) framework, which utilizes nine indicators to determine the Pressure indicator and three to assess the State indicator. All indicators were obtained using SNAP 9.0.0 software and were output using ArcGIS 10.7 software. The equations were applied using GIS 10.7 software. On the other hand, the Pressure Indicators were including of Digital Elevation Model (DEM), Global Environmental Monitoring Index (GEMI), Land Use/Cover (LULC), Normalized Difference Moisture Index (NDMI), Normalized Difference Water Index (NDWI), Soil Adjusted Vegetation Index (SAVI), Road network and Railway network, Land Surface Temperature (LST). The state indicators were included from Fractional Vegetation Cover (FVC), Normalized Leaf Area Index (LAI), and Normalized Difference Vegetation Index (NDVI). To ensure that each indication has a comparable weight and significance in the outcome, all indicators were rescaled and normalized from 0 to 1. The overall methodology is explained in [Figure 2](#).

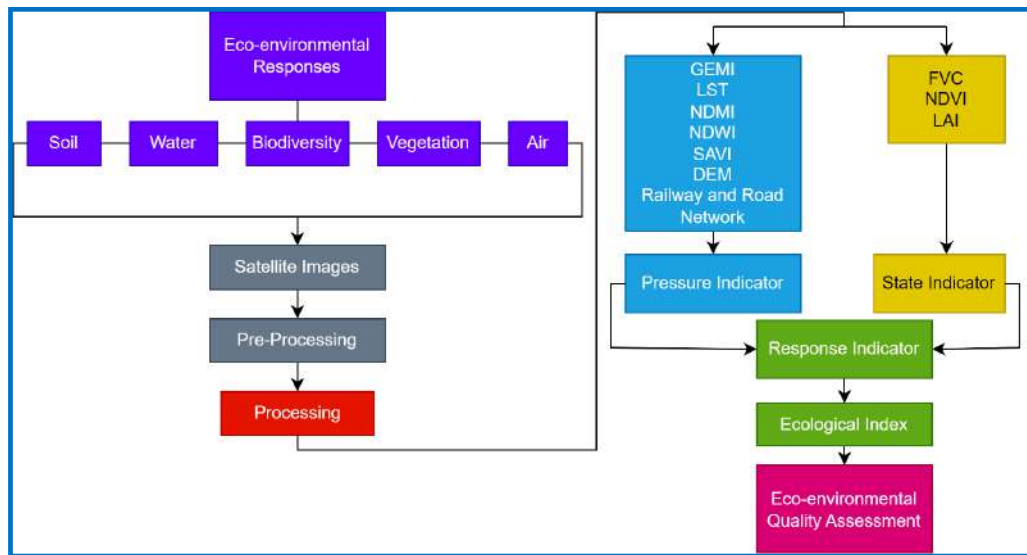


Figure 2: The Overall Methodology

2.3.1. Indicators used:

1. Global Environmental Monitoring Index (GEMI): GEMI is a nonlinear vegetation index derived from satellite data used for global environmental monitoring, more resistant to atmospheric influences than NDVI, ranging from 0 to 1, where 0 indicates the absence of plant cover and 1 indicates complete vegetation cover on the ground [12]. **Figure 3** shows the GEMI.
2. Fractional vegetation cover (FVC): is a crucial measure for understanding soil erosion, climate change, and ecosystem balance. It measures plant arrangement on Earth's surface, ranging from 0 to 1, with 0 indicating no vegetation and 1 indicating complete cover [13]. **Figure 4** shows the obtained FVC.
3. Leaf Area Index (LAI): is a dimensionless number used to characterize plant canopies, ranging from 0 to 10, indicating the presence of dense conifer forests [14]. **Figure 5** shows the LAI.
4. Normalized Difference Moisture Index (NDMI): This index is not standardized. The index, ranging from -1 to 1, is easily comprehensible and indicates a higher level of vegetation health and density [15]. The dry matter content of leaves is highly correlated with the NDMI indices [16]. **Figure 6** shows the NDMI.
5. Normalized Difference Vegetation Index (NDVI): Sensor data is used to measure plant density and health, with values ranging from -1 to 1, indicating greater vegetation abundance and density [15]. The index reduces or deletes characteristics with low red light and near-infrared reflectance, like water, while amplifying those with high NIR reflectance and lower red light reflectance, like terrestrial vegetation [17]. **Figure 7** shows the NDVI.

6. Normalized Difference Water Index (NDWI): Sensor data quantifies vegetation density and health using near-infrared and red wavelengths. Higher values indicate healthier vegetation. NDWI can provide information on liquid water content, although spectrum-matching techniques aren't suitable for determining vegetation water content [18]. **Figure 8** shows the NDWI.

7. Railway and road network: Human activity impacts ecosystems by examining road and rail networks, infrastructure, and utility systems. Increased modifications indicate higher frequency of activity. Population density correlates with larger roads, facilitating more frequent transportation. Road data in the research region includes primary, secondary, residential, and local roadways [7]. The increase in traffic brought on by projects could be offset by further highway funding [19]. The estimated railway and road networks are shown in **Figure 9**, where (A) is the road network map and (B) is the railway network map.

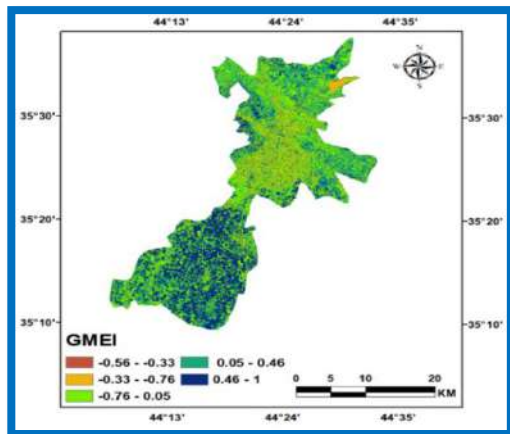


Figure 3: GEMI of Winter 2023.

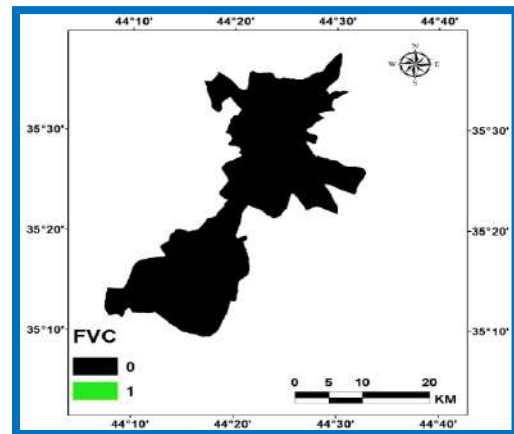


Figure 4: FVC of Winter 2023.

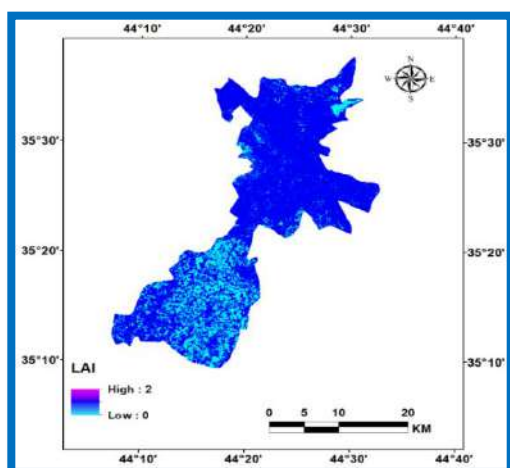


Figure 5: LAI of Winter 2023.

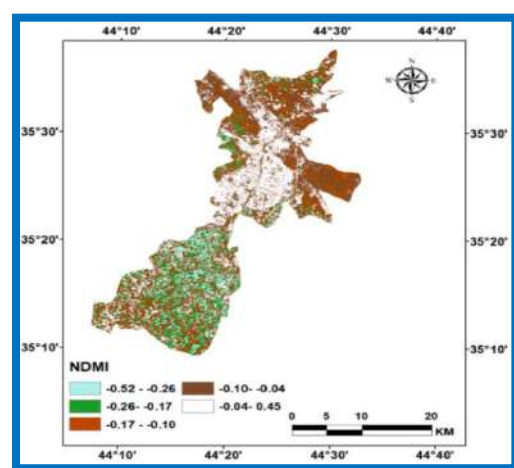


Figure 6: NDMI of Winter 2023.

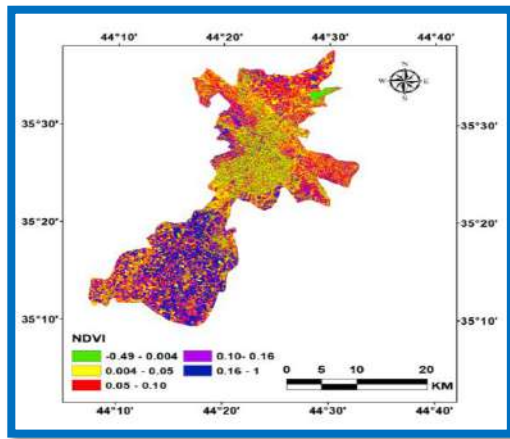


Figure 7: NDVI of Winter 2023.

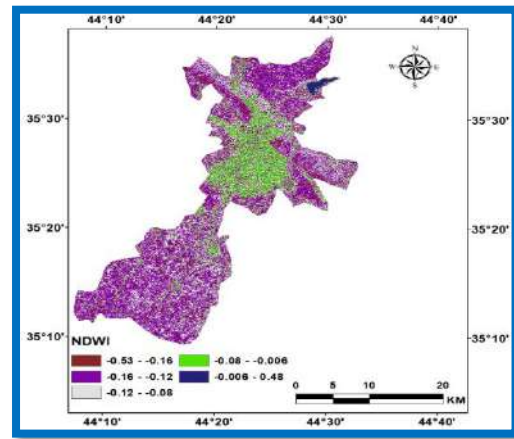
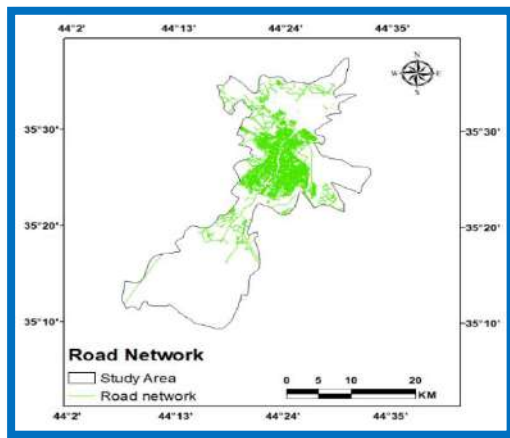
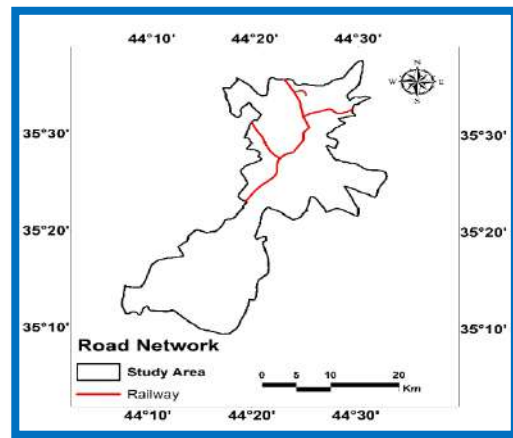


Figure 8: NDWI of Winter 2023.



(A) Road network



(B) railway network

Figure 9: Road Network and Railway Network

8.Land Use/Land Cover: Accurate land use and land cover data is crucial for urban planning, decision-making, population dynamics, and public health assessment [20]. Assessing land use and vegetation cover is crucial for urban planning and policy formulation due to growing structures and population spread in metropolitan areas [21]. Land inventories involve land use and land cover, crucial in climate models. Land use describes ecosystem function, social, economic, and cultural utility, requiring analysis of socioeconomic activities in the location [22]. The Support Vector Machine (SVM) classification method was used to obtain classes represented by urban areas, water bodies, barren areas, agricultural areas, and vegetation. The classification is shown in Figure 10.

9.Soil-Adjusted Vegetation Index (SAVI): Land inventories involve land use and land cover, crucial in climate models. Land use describes ecosystem function, social, economic, and cultural utility, requiring analysis of socioeconomic activities in the location [23]. It has significantly improved the development of global models that accurately describe dynamic soil-vegetation systems using remotely sensed data [24]. Figure 11 shows the SAVI.

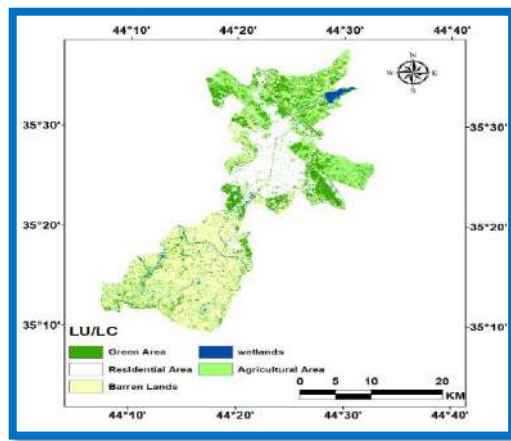


Figure 10: LULC of Winter 2023.

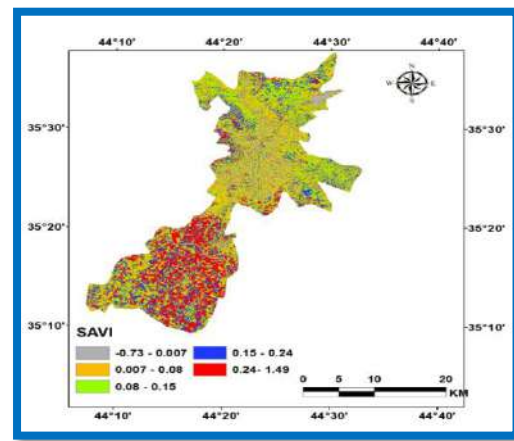


Figure 11: SAVI of Winter 2023.

10 . Digital Elevation Model (DEM): A digital elevation model (DEM) is a crucial spatial resource in GIS, representing terrain through a collection of digital data indicating ground elevation (spot height) [25]. Topography significantly impacts water balance in catchments, affecting surface and subsurface runoff, water movement, and routes. Fully distributed hydraulic and hydrological models use topography, represented by the Digital Elevation Model, to establish bathymetry [26]. The DEM that was used in this study is represented in Figure 12.

11. Land Surface Temperature (LST): Remotely sensed land surface temperature (LST) is intriguing for biological and environmental purposes due to challenges in weather observatories, field surveys, and data interpolation [27]. Land surface temperature (LST) is a crucial parameter in land-surface models, influencing turbulent heat exchanges and long-wavelength radiation at the ground-atmosphere interface, affecting aridity, soil moisture, and evapotranspiration [28]. It is directly correlated with the development and distribution of vegetation and the cycle of evaporation of surface water resources [29]. Figure 13 shows the obtained LST.

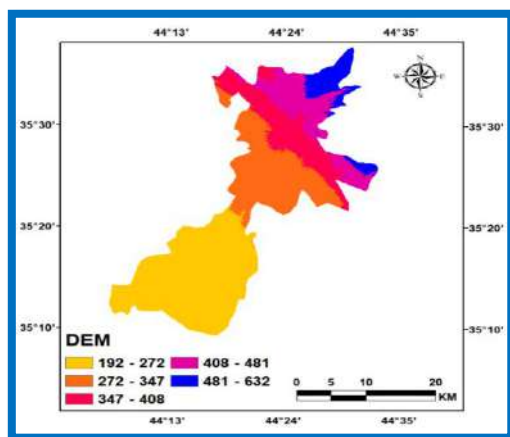


Figure 12: Digital Elevation Model (in meters) of Kirkuk City and Taza District

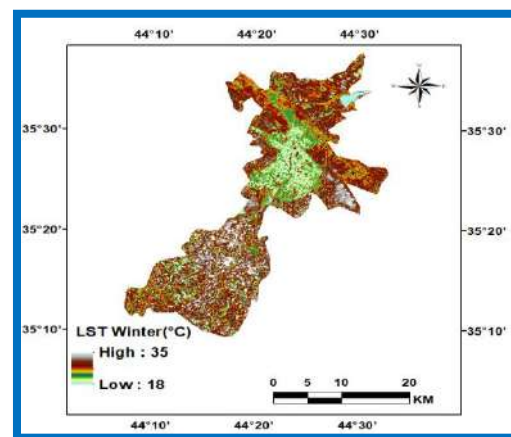


Figure 13: LST of Winter 2023.

2.3.2. PSR framework: The Pressure-State-Response concept explains how human activities impact the environment, providing more comprehensive information than two-dimensional indicator sets, using three categories: Pressure Indicator, State Indicator, and Response Indicator [30]. The dynamic and systematic interactions between the economic, social, and ecological environments can be reflected in the three dimensions of the PSR framework [31]. Depending on the concerns or progress that need to be looked at, a PSR framework's indicators can be chosen accordingly. There wouldn't be a standard set of indicators, and the choices might differ depending on the nation or the location. Data accessibility is still another crucial factor [32].

2.3.2.1. Pressure indicator (PI): Human activity's impact on ecosystem health is described using pressure indicators like social and resource demands [33]. the PI value from standardized data ranges from 0 to 1, giving each indicator in the research region equal weight, as shown in Eq. (1) [7].

$$PI = \frac{GEMI + SAVI + NDMI + LULC + Road + Rail + LI + NDWI + DEM}{9} \quad (1)$$

2.3.2.2. State indicator (SI): State indicators accurately represent an ecosystem's current health status by assessing its robustness, structure, and adaptability [33]. Healthy natural phenomena, such as NDVI, LAI, FVC, forests, mangroves, wetland wetlands, and waterbodies, are generally indicative of healthy ecosystems. All parameters were first standardized from a range of 0 to 1 and then given equal weight as per Eq. (2) to generate the state indicator. Higher SI values suggest improved ecological circumstances, while lower values indicate deteriorating ecological conditions [7].

$$SI = \frac{(NDVI + LAI + FVC)}{3} \quad (2)$$

2.3.2.3. Response indicator (RI): Response indicators depict an ecosystem's response to changes in its overall well-being, encompassing human activities and internal processes [33]. Response indicators indicate high-pressure conditions, indicating ecological disruption, while low reaction indicators suggest stable conditions with minimal changes due to lower demand, indicating sustainable development and controlled ecology, contrasting with elevated reactions suggesting significant environmental changes. RI can be calculated from the Eq. (3) [7].

$$RI = PI - SI \quad (3)$$

2.3.2.4. Calculation of EI: The ecological index significantly influences ecological quality assessment due to its consistency with average values within intervals, and normalizing it during calculation is crucial due to inconsistent dimensions [34]. EI can be calculated from Eq.

(4), where EI is an ecological indicator, and w and c represent the weight and standardized data [7].

$$EI = \sum_{i=1}^n W * C \quad (4)$$

By using all the ecological response parameters, the ecological indicator was calculated in this paper from Eq. (5).

$$EI = w(environment) + w(climate) + w(soil\ moisture) + w(greenness) + w(LCLU) \\ + w(artificial\ features\ \&\ energy) + w(water\ content) \\ + w(landscape) \quad (5)$$

Where the Environmental parameter refers to the global environmental monitoring index (GEMI); Climate parameter: Soil moisture: soil adjusted vegetation index (SAVI) and normalized difference moisture index (NDMI); Greenness: normalized difference vegetation index (NDVI), leaf area index (LAI), fractional vegetation cover (FVC); Land use/land cover: LULC change; Artificial features and energy: Road network, Railway network; Water content: normalized difference water index (NDWI); Landscape: digital elevation model (DEM).

The study analyzes the Eco-environmental quality assessment (EEQ) using the pressure-state-response (PSR) approach and describes the Ecological Index (EI) of winter 2023.

3. Results:

3.1.General assessment of PSR

3.1.1. Pressure indicator: The PI is a measure of atmospheric pressure in urbanized areas, with high levels observed in developed areas. The central region of Kirkuk city experiences high pressure, while surrounding villages experience mild pressure due to lower socio-economic activities. The PI is positively correlated with population size and negatively correlated when population decreases as shown in [Figure 14](#).

3.1.2. State indicator: The SI is mostly derived from the measurements of FVC, NDVI, and LAI. As a result of the humid winters in Kirkuk City, the vegetation was in excellent condition. The southwestern portion of the research region has dense vegetation, resulting in comparatively higher NDVI values towards the south. Given that both FVC and LAI serve as indicators of robust vegetation, they exhibited a similar NDVI pattern. The ultimate SI map, depicted in [Figure 15](#), exhibits the combined influence of all vegetative indices. The SI exhibits consistently high values in both seasons.

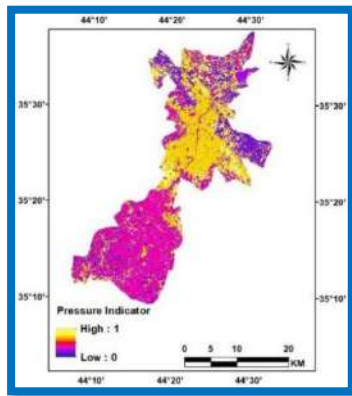


Figure 14: PI of Winter 2023.

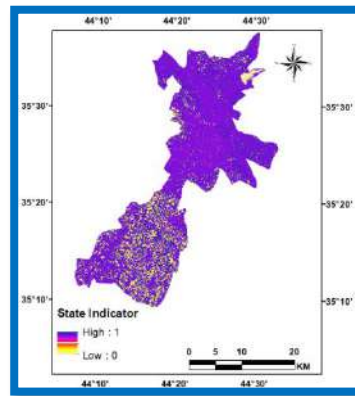


Figure 15: SI of Winter 2023.

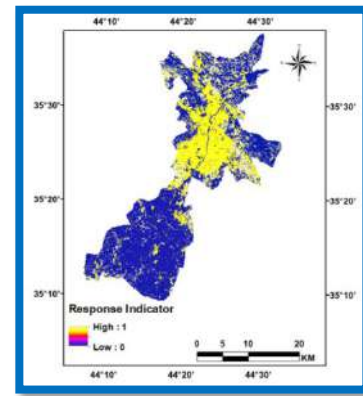


Figure 16: RI of Winter 2023.

3.1.3. Response indicator: Under significant anthropogenic strain, ecological conditions that are weaker, unhealthy, and unstable are indicated by strong reaction indicators, and conversely. Elevated RI values are symptomatic of heightened natural/human pressure and socioeconomic activity, such as industrial expansion, farming, and urban growth, which can lead to ecological disruption. Low RI values indicate less human involvement in ecosystems, including green fields, aquatic bodies, and remote areas far from metropolitan centers. According to the RI maps, the ecological status is consistently progressing, as seen in Figure 16. The study suggests that the southern part of the area possesses more favorable ecological traits in comparison to its central section. The data suggests that the resilience index (RI) for the agriculture and cultivation sectors was lower compared to the RI for socioeconomic activity locations.

3.2. Ecological Index: Elevated EI levels imply an ecological ecosystem that is strong and expanding, while lower values indicate the opposite. Figure 17 shows a study region with a moderate to high EI value. The northeastern and eastern areas of the research region exhibit slightly superior conditions compared to other sections, as they display the greatest values concentrated around Khasa and in close proximity to Sulaymaniyah city towards the east. Human activity intensifies in the center region of the study area. Consequently, the central area of Kirkuk City has the lowest values spread out over a significant distance. The southern and southwestern regions of the research area, where Taza is situated, exhibit a moderate EI value, as depicted in Figure 17. The spatial distribution of EI maps indicates that the regions in close proximity to water bodies had exceptional EI conditions, while the adjacent area displayed a range of EI conditions from excellent to moderate. There were several industrial and residential regions characterized by suboptimal ecological conditions. The northern part of the research area has a high level of ecological condition, ranging from good to exceptional. In contrast, the southern part of the area shows a moderate level of ecological condition, ranging from good to

acceptable. **Table 2** displays assessments and their respective regions derived from **Figure 17**. In contrast to the Taza district, the central area of the research region and Kirkuk City exhibit ecological conditions ranging from fair to bad. This highlights the urgent need for an intervention by a responsible and responsive ethics commission to address this significant problem.

Table 2: Evaluations and their Corresponding Areas.

Evaluation	Corresponding area (km ²)
Extreme excellent – excellent	229.3
Excellent – good	142.5
Good – fair	165.93
Fair – poor	97.36
Poor – bad	44.65

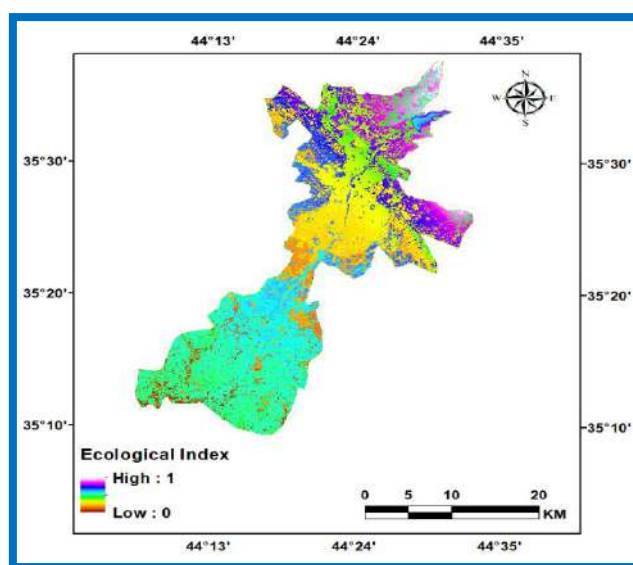


Figure 17: EI of Winter 2023.

4. Discussion

This research aims to identify and measure the ecological index to support the Environmental Quality Evaluation (EEQ) in Kirkuk City, Iraq. The Environmental Index (EI) is considered the most precise measure for evaluating the environmental state. GIS and remote sensing technologies are used to monitor environmental quality in areas like Kirkuk City, Iraq. The study uses time series remote sensing satellite data and the PSR framework to develop 12 indicators related to environmental concerns. The assessment considers land use/cover change, human and natural pressure, the environment's condition, and ecosystem health. The study also examines the role of the vegetative ecosystem in reducing pressure indicators and protecting the environment. The study identifies areas that have been safeguarded by the government to conserve the environment and minimize human activities. The findings are significant for

NGOs, government policymakers, and individuals interested in sustainable development due to their wide-ranging applications.

5. Conclusions

This research utilizes GIS and remote sensing data to assess the eco-environmental status. It achieves this by calculating the primary ecological indicators within the PSR framework in order to control the EEQ. The EI effectively evaluates the condition of the study region by employing a range of indicators that measure different influences on the environmental system. The study identifies that dryness and high temperatures have detrimental effects on the environment, while indicators of greenness and wetness have a positive impact. The improvement in EEQ (Environmental Quality Index) is dependent on the positive values observed in natural responses. Human activity, especially social and economic activities, significantly impacts the ecological health by causing harm to the system.

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Synthesis, Characterization, and CO₂ Capture Application of Cu(II)-paracetamol complex

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Keywords: Cu(II), Paracetamol, CO₂ Capture, Metal Complexes , Tridentate Ligand.

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Abstract:

A cu(II)-paracetamol complex was synthesized and characterized using UV-Vis and FTIR spectroscopy and melting point analysis. The ligand paracetamol acted as a tridentate chelating agent, coordinating through the hydroxyl, carbonyl, and amine groups. The complex was found to be more soluble in DMSO. Job's method of continuous variation suggested a 1:2 metal-to-ligand stoichiometry. The synthesized Cu(II) complex was evaluated for CO₂ adsorption performance, demonstrating promising uptake capacity due to the strong affinity of the metal center for CO₂ molecules.

Keywords: Cu(II), Paracetamol, CO₂ Capture, Metal Complexes, Tridentate Ligand.

تحضير، توصيف، وتطبيق احتجاز ثاني أكسيد الكربون لمعقد النحاس-باراسيتامول

الخلاصة:

تم تحضير وتوصيف معقد-Cu(II) باراسيتامول باستخدام أطيف الأشعة فوق البنفسجية-المرئية (UV-Vis) وتقنية الأشعة تحت الحمراء (FTIR) بالإضافة إلى تحليل نقطة الانصهار. عمل الباراسيتامول ك ليغند ثلاثي السن، منسقاً من خلال مجموعات الهيدروكسيل والكربونيل والأمين. وُجد أن المعقد أكثر ذوبانية في DMSO. أظهرت طريقة جوب للتغير المستمر أن نسبة المعدن إلى الليغند هي ١:٢. تم تقييم المعقد المحضر من Cu(II) من حيث أدائه في امتصاص غاز ثاني أكسيد الكربون، حيث أظهر قدرة امتصاص واعدة بسبب قوة التآلف العالية بين مركز المعدن وجزيئات CO₂.

الكلمات المفتاحية: Cu(II)، باراسيتامول، احتجاز ثاني أكسيد الكربون، المعقدات الفلزية، ليغند ثلاثي السن.

1. Introduction:

Paracetamol (acetaminophen) is a commonly used analgesic and antipyretic drug that has also gained attention in coordination chemistry [1] [2]. Paracetamol exhibits important ligand properties due to the presence of hydroxyl (-OH), amine (-NH₂), and carbonyl (C=O) groups, making it suitable for coordination with transition metals [3] [4]. These groups act as donor sites in chelation, forming stable complexes with various metal ions.

Copper(II) ions are particularly valuable in coordination chemistry due to their flexible geometry, strong Lewis acidity, and redox activity. Cu(II) complexes have shown monumental applications in catalysis, antibacterial agents, and environmental treatment [5]. When coordinated with multidentate ligands such as paracetamol, copper ions often adopt square planar or distorted octahedral geometries, depending on ligand field strength [6].

Copper(II) complexes of various ligands have demonstrated wide biology and catalytic applications due to the favorable redox behavior of Cu(II) ions. Despite studies on Cu(II)-paracetamol complexes in biomedical and catalytic areas, their potential role in environmental applications- especially CO₂ capture- remains largely unexplored. This represents a critical research gap, especially given the global need for sustainable and cost-effective carbon dioxide sequestration technology [7].

Traditional adsorbents such as zeolites, activated carbon, and metal-organic frameworks (MOFs) are widely studied for CO₂ capture. However, challenges such as high cost, synthetic complexity, or recovery difficulty limit their broader usages [8] [9] [10]. Recent literature has shown the importance of exploring novel coordination complexes as functional CO₂ adsorbents. The strong Lewis acidity of Cu(II) and the formation of a polar surface upon complexation

make such materials promising for binding CO₂ through physisorption or chemisorption mechanisms [11] [12][13].

The objective of this study is to synthesize and characterize a Cu(II)-paracetamol complex, investigate its coordination manner, and evaluate its performance in CO₂ capture. A schematic representation of the expected coordination structure will later be provided in the Discussion section to clarify the binding mode between copper and paracetamol. The importance of this work demonstrates a simple, accessible, and functional coordination complex stemming from a pharmaceutical ligand for environmental CO₂ capture under mild conditions.

2. Material and methods:

Pure paracetamol (C₈H₉NO₂, 99% purity), melting point:(169°C) was obtained from Kendy Pharmaceuticals, Cu(NO₃)₂.6H₂O analytical grade, methanol, and distilled water were used as solvents. The copper complex was synthesized by mixing 0.01 mol of Cu(NO₃)₂.6H₂O dissolved in 100 ml of distilled water and 0.02 of paracetamol dissolved in 100 ml of methanol. The reaction mixture was mixed and stirred for 1 hour at room temperature (~25°C) and subsequently heated at 60 °C for 2 hours on a hot plate until all the solution was evaporated. A greenish-dark precipitate was formed, filtered using a Bucher funnel, and dried at ambient temperature. The obtained complex was characterized for thermal stability, color, and spectroscopic signatures [14].

FTIR spectra were recorded in the range of 400 -4000 cm⁻¹ using KBr pellets (1:100 sample-to-KBr ratio). Spectra were collected for both the free ligand and the synthesized complex to identify shifts in functional group frequencies due to metal coordination. Additionally, UV-Visible spectra were obtained in the range of 200 -800 nm using methanol as solvent. Absorbance changes between free ligand and complex solutions were evaluated to confirm electronic transitions and coordination [15] [16]. Moreover, the Job's method of continuous variation was used to determine metal-to-ligand stoichiometry, where absorbance values at a fixed wavelength were plotted against mole fractions of paracetamol [17]. Gravimetric CO₂ adsorption experiments were conducted within a temperature range from 25 to 60 °C under 1 atm pressure, in which adsorbent mass and exposure time were kept constant, and uptake values were calculated in mmol/g. Additionally, as the entire process was done in the solid phase, PH control was not necessary.

3. Results:

The coordination of paracetamol with the Cu(II) ion was elucidated through Fourier Transform Infrared Spectroscopy (FTIR) and Ultraviolet-Visible (UV-Vis) Spectroscopy,

which are key techniques for confirming metal-ligand interactions and electronic transitions in coordination compounds [18]. Moreover, the Cu(II)-paracetamol complex showed observable CO₂ adsorption performance under room and increased temperatures. At 25 °C and 1 atm, the complex demonstrated a CO₂ uptake of 1.22 mmol/g, indicating moderate affinity toward CO₂. As the temperature increased, the adsorption capacity decreased to 0.95 mmol/g at 40 °C and 0.59 mmol/g at 60 °C, compatible with the exothermic nature of physisorption. The adsorption manner followed the Langmuir isotherm model, indicating monolayer surface coverage and confirming physical adsorption as the main mechanism [19] [20] [21]. The trend shows that the CO₂ capture is favorably achieved at lower temperatures. Compared to the standard adsorbents such as activated carbon and zeolites, the Cu(II)-paracetamol performs within a suitable range, highlighting its importance as an easy synthesis and a cost-effective material for CO₂ capture applications.

4. Discussion

The FTIR spectrum of free paracetamol revealed characteristic vibrational modes assigned to its functional groups for its structural identity and purity. The broad band around 3325 cm⁻¹ is assigned to O-H stretching, which is typically appears in this region due to hydrogen bonding. Additionally, a distinct sharp band at 3163 cm⁻¹ corresponds to N-H stretching of the amine group, confirming the presence of an acetamide moiety in the molecule.

Another prominent peak observed at 1650 cm⁻¹ is attributed to C=O (carbonyl) stretching of the amide functionality. This band is generally strong and sharp, supporting the presence of a conjugated amide system. The C-N bending and N-H bending vibrations typically appear between 1500-1580 cm⁻¹ and indeed several medium-intensity bands are seen in that region, reinforcing the assignment of an acetamide structure.

Furthermore, peaks observed in the region of 1240-1020 cm⁻¹ are consistence with C-O stretching vibrations from the phenolic group, and the out-of-plane C-H bending of the aromatic ring appears in the 750-840 cm⁻¹ range.

These spectral features collectively confirm the presence of hydroxyl, amide, and aromatic functionalities in the paracetamol molecule. **Figure 1** shows the FTIR spectrum of free paracetamol, clearly indicating all characteristic absorption bands discussed.

Upon complexation with Cu(II), significant shifts and changes in these absorption bands were observed in the FTIR spectrum of paracetamol, indicating coordination through multiple donor sites. Specifically, the O-H stretching band was shifted from 3325 cm⁻¹ to approximately 3562 cm⁻¹ in the complex, indicating involvement of the phenolic -OH group in direct

coordination to the metal center, albeit without full deprotonation. Furthermore, the N-H stretching band at 3163 cm^{-1} , which was clearly visible in the free paracetamol spectrum, completely disappeared from the spectrum of the Cu(II) complex. This disappearance supports the coordination of the amide nitrogen to the metal center via the lone pair donation, suggesting chelation through the nitrogen atom.

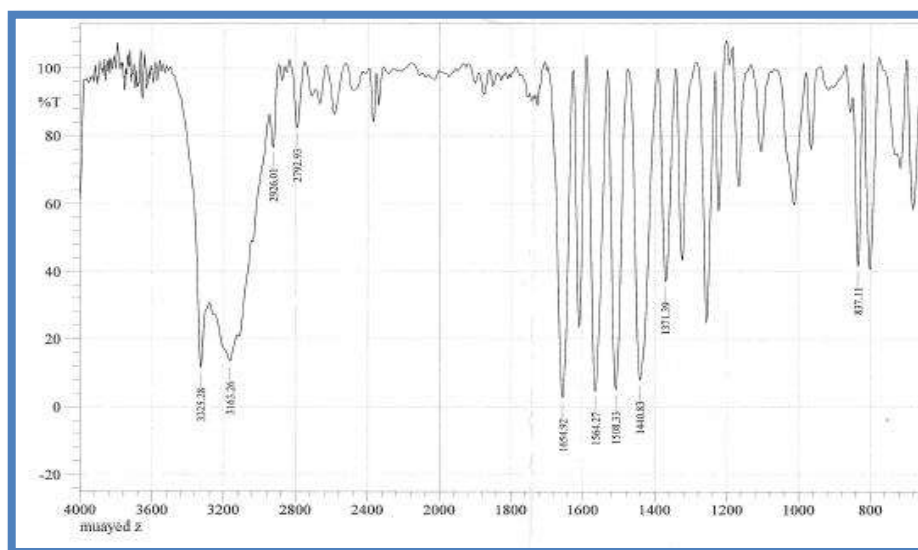


Figure 1: FTIR spectrum of pure paracetamol, showing characteristic absorption bands of functional groups present in the molecule

The C=O band, typically observed at 1650 cm^{-1} , was significantly shifted to a broader, less intensity region between $1614\text{--}1548\text{ cm}^{-1}$. This red shift in the carbonyl vibration is characteristic of coordination via the carbonyl oxygen, which weakens the C=O bond due to electron delocalization toward the metal ion. Such behavior is well-known in the metal-amide complexes binding modes. Additional spectral changes were observed in the low-frequency region ($<600\text{ cm}^{-1}$) where new bands emerged that were absent in the spectrum of free paracetamol. These are assigned to M-O and M-N stretching vibrations, further confirming the formation of a metal-ligand coordination network.

Altogether, these spectral changes- namely the shift or disappearance of O-H, N-H, and C=O bands, and the appearance of metal-ligand vibrational modes - strongly confirm that paracetamol behaves as a tridentate ligand, binding through its hydroxyl oxygen, amide nitrogen, and carbonyl oxygen atoms. This coordination configuration forms a stable chelate with Cu(II), likely resulting in a six-membered chelate ring. **Figure 2** presents the FTIR spectrum of the Cu(II)-paracetamol complex, clearly showing the discussed changes and confirming successful complex formation.

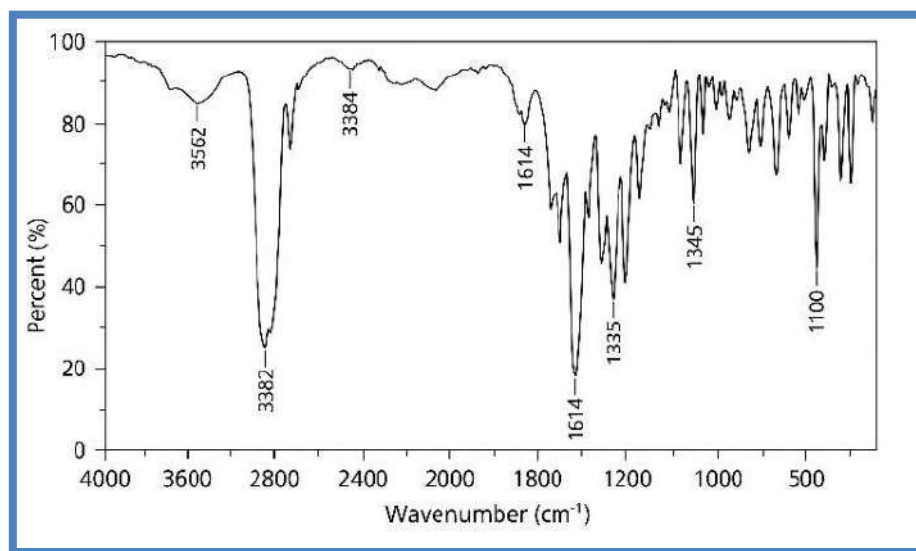


Figure 2: FTIR spectrum of pure Cu(II)-paracetamol complex, showing characteristic absorption bands. Key peaks indicate the presence of functional groups involved in complexation.

UV-Vis absorption spectrum of the free paracetamol ligand exhibited a prominent peak around ~243 nm, shown in **Figure 3**, which is associated with $\pi \rightarrow \pi^*$ transitions of the aromatic ring system. These transition are characteristic of conjugated systems and provide evidence of the electronic configuration and aromatic character of the ligand. The band reflects the presence of delocalized π electrons in the phenyl ring of paracetamol and is often used as a fingerprint region for aromatic compounds.

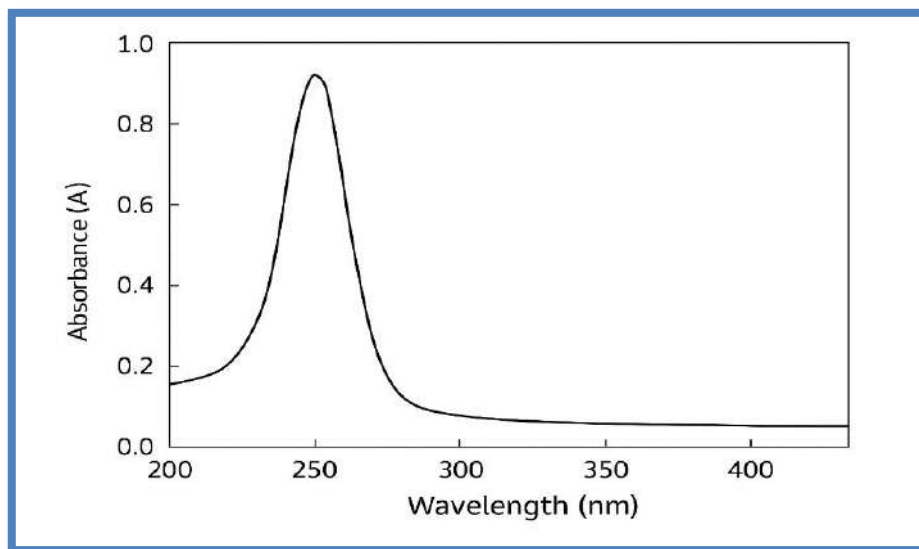


Figure 3: UV-Visible absorption spectrum of free paracetamol.

However, upon complexation with Cu(II), the absorption maximum (λ_{max}) was red-shifted to approximately 340 nm, indicating a change in the electronic environment of the ligand due to metal coordination as depicted in **Figure 4**. This bathochromic shift is characteristic of $n \rightarrow \pi^*$ transitions and ligand-to-metal charge transfer (LMCT) interactions, suggesting a strong

interaction between the lone pair electrons (particularly on oxygen and nitrogen atoms) and the Cu(II) metal center. Such transitions are indicative of delocalization of electron density across the metal-ligand framework, which is common in chelated systems involving transition metals. The presence of a d-d transition band observed near 580 nm was also observed in the visible region, often contributing to the distinctive greenish coloration of the complex. These electronic transitions are consistent with a distorted octahedral or square planar geometry, commonly adopted by Cu(II) complexes, depending on the ligand field strength and donor set.

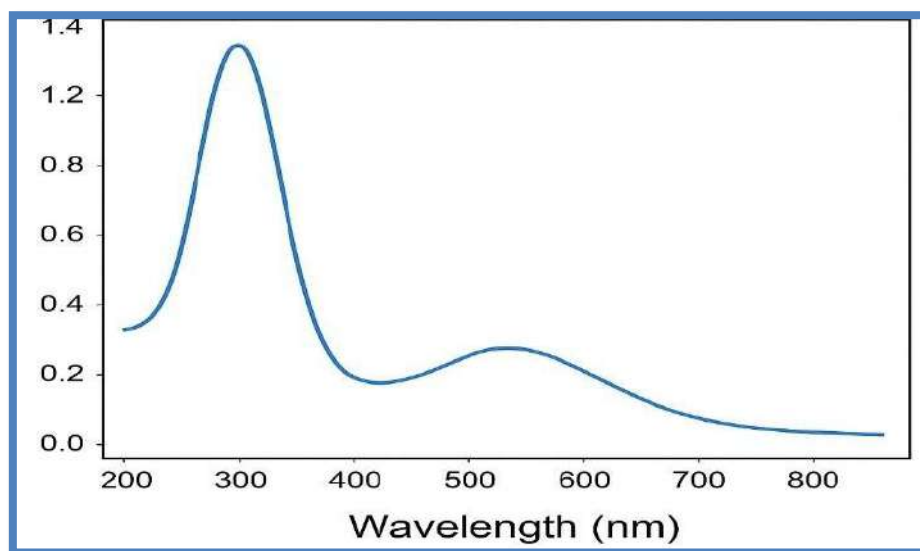


Figure 4: UV-Visible absorption spectrum of the copper-paracetamol complex recorded in the range of 200-800 nm

The plot of continuous variations (Job's method) provides important insights into the stoichiometry and complexation behavior between copper (II) and paracetamol in solution. The absorbance is plotted against the mole fraction of paracetamol to determine the binding ratio between the central ion and the ligand shown in **Figure 5**.

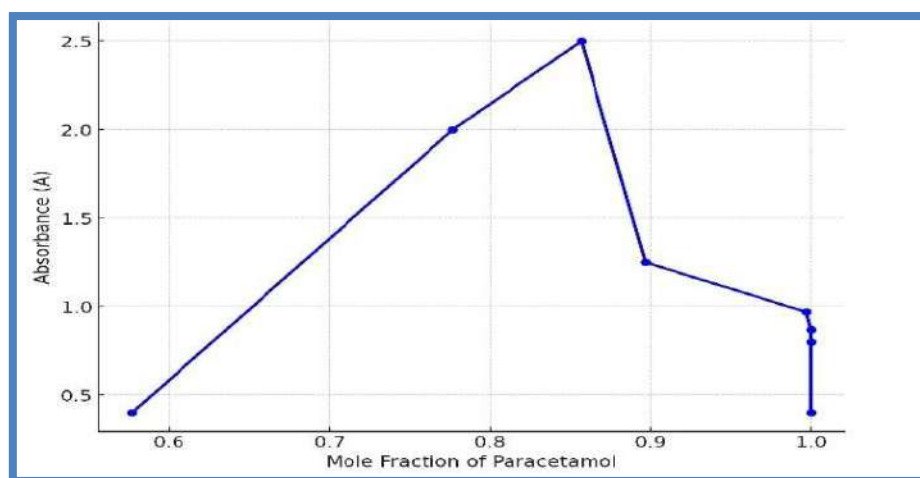


Figure 5: Jon's method plot showing the variation in absorbance of the Cu(II)-paracetamol complex with mole fraction of paracetamol, indicating a 1:2 metal-to-ligand stoichiometry at maximum absorbance (0.857 mole fraction)

The curve in **Figure 5** shows a distinct profile characterized by a gradual increase in absorbance as the mole fraction of paracetamol increases, reaching a maximum at about 0.857 mole fraction, followed by a successive decrease. The maximum absorbance peak is associated with the composition at which the molar ratio of approximately 1:2 metal-to-ligand ratio, suggesting that each copper ion binds with two molecular units of the ligand. Such stoichiometric data are consistent with prior works on Cu(II)-paracetamol complex [22] [23], and the proposed coordination environment is further supported by FTIR and UV-Visible. Spectroscopic findings. The ligand-to-metal ratio indicates a strong interaction between the phenolic, amide, and carboxylate groups of paracetamol, possibly forming an octahedral coordination environment. To support the Job's plot data, **Figure 6** illustrates a schematic bonding diagram of the proposed Cu-paracetamol complex, confirming the tridentate configuration. This structural presentation is crucial for correlating spectroscopic changes with the complex geometry.

The decline in absorbance beyond the 0.857 mole fraction indicates that excess paracetamol does not contribute to further complex formation. Instead, it may give rise to ligand saturation. This means that free paracetamol stays unbound in solution and does not produce the UV-Visible absorbing complex. Therefore, the overall absorbance is decreased due to the dilution of the complex concentration.

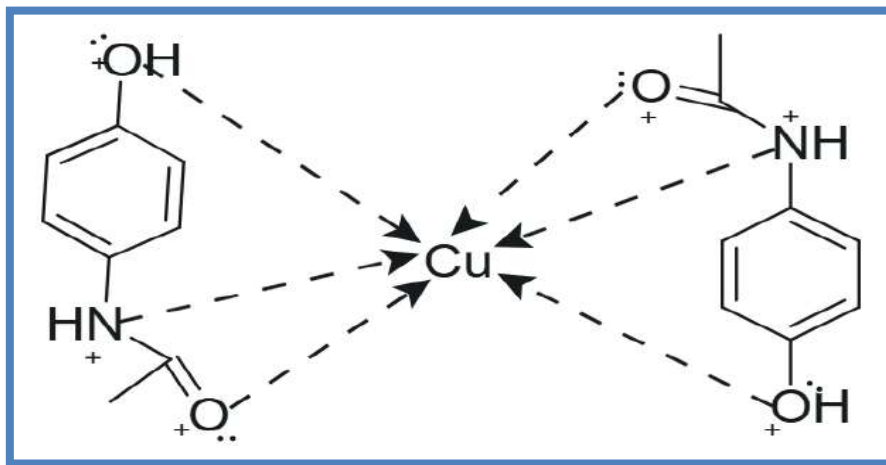


Figure 6: Proposed structure of the Cu(II)-paracetamol complex showing tridentate coordination by each paracetamol ligand through the phenolic-OH, amide nitrogen, and carbonyl oxygen, resulting in an octahedral geometry around the central Cu²⁺ ion

The adsorption measurements were done under 1 atm pressure for 60 minutes using 100 mg of the complex in a sealed chamber. The complex exhibited CO₂ uptake of 1.22 mmol/g 25°C and at 60 °C, the uptake values decreased to 0.59 mmol/g as shown in **Table 1**. The manner is consistent with the Langmuir model, which is expressed as $q_e = \frac{q_{max} KL Ce}{1 + KL Ce}$ [24] [25].

Where:

q_e is the amount of adsorbate at equilibrium (mmol/g)

$q_{\max}$ is the maximum adsorption capacity (mg/g)

C_e is the equilibrium concentration of adsorbate (mg/L)

and

KL is the Langmuir constant related to the affinity of binding sites (L/mg)

Table 1: CO₂ adsorption capacity of the Cu(II)-paracetamol complex at different temperatures under 1 atm pressure.

Temperature	Pressure	CO ₂ Uptake (mmol/g)
25 °C	1 atm	1.22
30 °C	1 atm	1.13
35 °C	1 atm	1.04
40 °C	1 atm	0.95
45 °C	1 atm	0.86
50 °C	1 atm	0.77
55 °C	1 atm	0.68
60 °C	1 atm	0.59

To demonstrate the thermal behavior of CO₂ adsorption on the complex surface, an Arrhenius-type plot of $\ln(q_e)$ versus $1/T$ (K⁻¹) was constructed using the experimental uptake values at different temperatures. A good linear relationship was noticed, suggesting the temperature dependence of adsorption follows the Arrhenius equation. The slope of the line was used to determine the adsorption energy, which was found to be **-16.89 kJ/mol**. This negative value confirms the exothermic nature of the adsorption process and is unique to physisorption, where a relatively weak van der Waals interaction is the main force between the adsorbate CO₂ and the adsorbent surface, as shown in **Figure 7**.

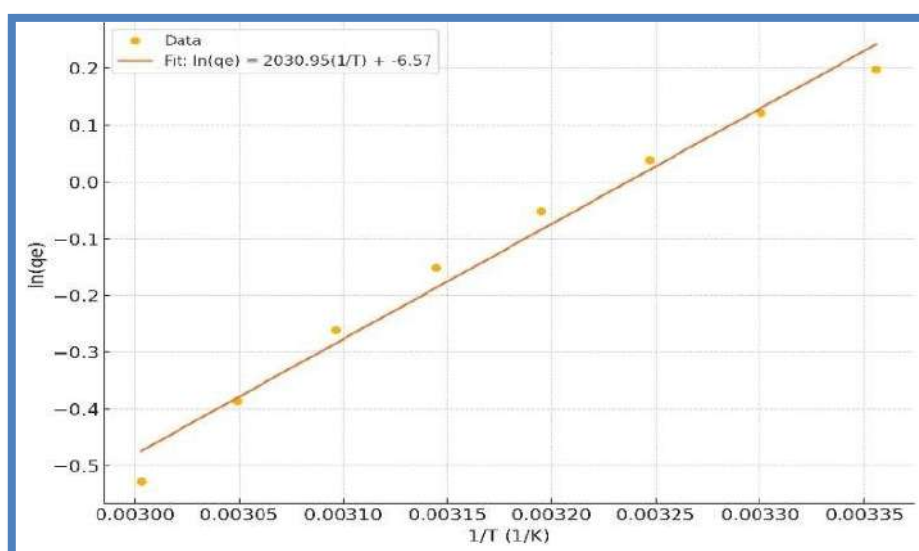


Figure 7: Arrhenius plot of $\ln(q_e)$ vs. $1/T$ for CO₂ Adsorption onto the complex

The Cu(II)-paracetamol complex demonstrated a CO₂ adsorption capacity of 1.22 mmol/g at 25 °C and 1 atmosphere, which is a significant result when compared with the class of molecular coordination complexes. This uptake indicates that the synthesized material possesses a moderately high affinity for CO₂ under ambient conditions, suggesting favorable thermodynamic and kinetic interaction between the gas molecules and the adsorbent. From a coordination chemistry perspective, the Cu(II) center acts as a Lewis acid with an open coordination site or labile positions in the ligand field. The adsorption mechanism is dominated by physisorption, likely driven by dipole-quadrupole interaction between the polar Cu(II) center and linear CO₂ molecules. The presence of functional groups such as hydroxyl(-OH), carbonyl (C=O), and amine (-NH₂) in the paracetamol ligand contributes additional Lewis basic sites, facilitating localized CO₂ adsorption via hydrogen bonding or acid-base. Furthermore, the relatively high uptake at 25 °C- where adsorption is typically thermodynamic favored- is consistent with physisorption behavior. The result also indicates that no significant steric hindrance occurs with the complex structure that would otherwise inhibit CO₂ accessibility to active sites. It is likely that the complex forms a supramolecular network or aggregates that retain free volume or interstitial voids, providing an accessible microenvironment for gas diffusion and retention [26]. In terms of performance benchmarking, the value of 1.22 mmol/g places this material in the same operational range as many non-porous adsorbents and functionalized organic compounds seen in Table 2. While it is not as high as metal-organic frameworks (MOFs), which often exceed 4.4 mmol/ g under similar conditions, it offers distinct advantages in terms of synthesis simplicity, stability in ambient environments, and potential biodegradability, which are crucial for low-cost and CO₂ management systems.

Table 2: Comparison of CO₂ uptake capacities at 25 °C for various adsorbent materials

Materials type	Uptake at 25 °C mmol/g
Amine-functionalized silica	1.0-3.5
Zeolites	2.5-3.0
MOFs (e.g, HKUST-1)	4.5-6.0
Activate carbon	0.8-2.0
Cu(II)-paracetamol complex	~1.22

5. Conclusions

This study successfully synthesized a cu(II)-paracetamol complex, characterized by FTIR, UV-Vis spectroscopy, and stoichiometry analysis. The spectral data confirmed coordination through the hydroxyl , amine, and carbonyl groups of paracetmol, forming a tridentate ligand. UV.Vis analysis showed red-shifted absorption, indicating metal-ligand interactions. The

complex exhibited a CO₂ uptake capacity of ~1.22 mmol/g under ambient conditions, suggesting potential as a cost-effective and environmental friendly CO₂ capture material. These results demonstrate the dual utility of the complex in both pharmaceutical coordination chemistry and environmental applications.

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Effect of the Structural and Electrical Properties of $\text{Bi}_{2-x}\text{Pb}_x\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ Superconductors with Partial Substitution of Lead by Bismuth

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Abstract:

This manuscript discusses the preparation of $\text{Bi}_{2-x}\text{Pb}_x\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ compounds by the method of solid-state reaction at the annealing temperature $850\text{ }^\circ\text{C}$. Under pressure of 8 ton/cm^2 with a presence of enough oxygen and these are considered as ideal conditions according to the previous researches in the preparation of electric superconductors of high degree temperatures the effect of partial substitution was on Lead (Pb) element in the Bismuth (Bi) element was studied to produce a compound with formula of $\text{Bi}_{2-x}\text{Pb}_x\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ and for different ratios of x to know the effect of partial substitution to form Bi2223 phase at the annealing time 72 hrs and we concluded from study of diffraction of x-ray that the compound kept its tetragonal structure, and critical temperature (T_c) degrees were obtained $T_c = 128\text{K}, 130\text{K}, 135\text{K}, 132\text{K}$, and that's at substitution ratios $x=0, 0.15, 0.25, 0.3$, respectively it was clear the best substitution ratios of pb in Bi was when $x=0.25$.

Keywords: Bismuth, Lead, Superconducting Properties, Solid State Reaction Method, Tetragonal Structure.

تأثير الخواص التركيبية والكهربائية لموصلات $Bi_{2-x}Pb_xBa_2Ca_2Cu_3O_{10+\delta}$ الفائقة عند الاستبدال الجزئي للرصاص بالبرزموت

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الخلاصة:

يدرس هذا البحث إعداد مركبات $Bi_{2-x}Pb_xBa_2Ca_2Cu_3O_{10+\delta}$ باستخدام طريقة التفاعل في الحالة الصلبة عند درجة حرارة تليد تبلغ 850 درجة مئوية، وتحت ضغط 8 طن/سم²، مع وجود كمية كافية من الأوكسجين، وتعتبر هذه الظروف مثالية بناءً على الأبحاث السابقة في تحضير الموصلات الفائقة الكهربائية ذات درجات الحرارة العالية. تمت دراسة تأثير الاستبدال الجزئي لعنصر Pb بعنصر Bi لإنتاج مركب بصيغة $Bi_{2-x}Pb_xBa_2Ca_2Cu_3O_{10+\delta}$ و لتراكيز مختلفة لـ x، وذلك لمعرفة تأثير الاستبدال الجزئي على تكوين الطور Bi2223 عند زمن تليد بلغ 72 ساعة. ومن خلال دراسة حيود الأشعة السينية (XRD)، تبين أن المركب يحافظ على صفته الرباعية (tetragonal)، وتم قياس درجات الحرارة الحرجة حيث كانت القيم 128K، 130K، 135K، 132K عند نسب استبدال $x = 0, 0.15, 0.25, 0.3$ على التوالي. وقد كان واضحاً أن أفضل نسبة استبدال لعنصر Pb بعنصر Bi هي عند $x = 0.25$.

الكلمات المفتاحية: البرزموت، الرصاص، خصائص الموصلية الفائقة، طريقة التفاعل في الحالة الصلبة، التركيب الرباعي.

1. Introduction:

When materials are cooled to a specific temperature, known as the T_c , they become superconducting, allowing electric current to flow without resistance and with no magnetic flux at all. This property is crucial for many applications in electronics, connectivity, and medical equipment. [1,2]. After the discovery of some ceramic materials with a temperature of more than 90 K, which were referred to as high-temperature superconductors, superconductors entered a new stage. The ability to use liquid nitrogen for cooling [3] and liquid nitrogen at a temperature of (77 K), which can be obtained readily and at an affordable price [4,5], and the discovery of these materials have mad.

High temperature superconducting materials, including the BBCCO system, have been identified and synthesize [6] is an abbreviation for HTSC, composed of element oxides (bismuth, barium, calcium, copper) with the typical chemical formula $(Bi_{2-x}Pb_xBa_2Ca_{n-1}Cu_nO_{6+\delta})$, where $(n = 1, 2, 3)$. It is of significant significance because of its high frequency, elevated temperature tolerance, and excellent chemical resistance to moisture. Consequently, significant efforts have been devoted to examining the preparation methods, treatments, and

properties of this system [7,8]. The superconducting system (BBCCO) exhibits a layered structure including copper oxide layers (CuO) with Critical temperatures (T_c) of 110, 80 and 10 K, respectively, at which the electrical resistance reaches zero ($R=0$) [9,10]. The superconducting and transition properties of high-temperature CuO compounds are very different [11,12]. The unsupported alternating conductivity at frequency, coupled with a low insulation constant at high frequencies (60 GHz) at room temperature, and the material's polarization may necessitate the use of frequency-dependent conductivity [12].

The discovery of high temperature superconducting material initiated a revolutionary advancement in industrial applications and materials science [13,14]. High temperature superconducting systems include Bi-2201, Bi-2212, and Bi-2223 [15], which are distinguished by their two-dimensional basis and 1-layered structure, comprising three phases: Bi-2201, Bi-2212, and Bi-2223. A final numeral for each phase denotes the quantity of CuO layers, which corresponds to the T_c (10K, 80K, and 110K); the latter signifies the temperature at which electrical resistance is zero ($R = 0$) [16]. The Bi-2223 phase is challenging to synthesize, although it possesses the advantage of being a single phase with the highest T_c (110 K) among 204 effects of partial substitution of silver and copper on the structural and electrical characteristics of $\text{Bi}_2\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ superconductors across three phases [17]. The characteristics of superconductors can be adjusted by incorporating or eliminating an element with varying ionic radius and bonding properties, and the enhancement or decline in superconducting properties is contingent upon the attributes of the added or substituted elements that differ in radius and bonding. The majority of the research concentrates on enhancing the morphology and characteristics of (Bi-2223) through substitution studies [18-20]. This current work discusses the temperature-dependent electrical characteristics and structural properties of the $\text{Bi}_{2-x}\text{Pb}_x\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ samples and at concentrations $x=0, 0.15, 0.25$ and 0.3 .

2. Material and methods:

The molecular weights of these materials relative to the weight of the element in each of the base material and the compound (sample) to be generated were used to calculate the weight ratios of the materials that contribute to the creation of the $\text{Bi}_{2-x}\text{Pb}_x\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ compound.

The oxides and carbonates are weighed out, combined, and then put into granules. These materials are then ground finely for 30 minutes using an agate grinder so that the mixture becomes homogeneous. An isopropanol solution is added during the grinding process to prevent falling or losing powder during the grinding process. The isopropanol alcohol is then removed from the grinder by placing it in an electric oven set between (50°C , 60°C).

These powders are put in a convection oven, which is heated to 850°C and heated at a pace of 120°C per hour in an air-saturated environment. This form is kept at (850 C) for (12) hours before being lowered to allow it to cool to a specific temperature. The model is taken out of the furnace at room temperature with a cooling rate of 30°C/hr and temperature control using a thermocouple. To prevent vaporization and loss, the powder is then combined and continuously processed for another 30 minutes. The ester of is solution is then added. 50°C, 60°C).

The powder is then manufactured under pressure (8 Ton/cm) in the form of tablets. These tablets had (12 mm) diameters and ranged in thickness from (0.8 mm) to (1.2 mm). These discs are put in an electric oven, where the temperature is progressively increased to (850 C°) at a pace of (120 C° /hr) and held there for 12 hours before being gradually lowered at a rate of (30 C°). After acquiring the samples that were made in the shape of tablets from the previous sentence, the process of heating and cooling takes place in an atmosphere that is saturated with oxygen until it reaches room temperature. This process is known as sintering. The prepared samples are obtained as tablets, which are then placed in an electric oven and heated to (600 C°) at a rate of (120 C° /hr) from room temperature. The sample is then kept at this temperature for (12) hours before the temperature is raised. The oven heats up to 850 degrees Celsius at a rate of 120 degrees Celsius per hour, where it stays for 24 hours in an oxygen-rich environment. The model then cools to 600 degrees Celsius at a rate of 30 degrees Celsius per hour, where it stays for another 12 hours before being cooled to room temperature at a rate of 60 degrees Celsius per hour to room temperature. Tablet-shaped samples are obtained, and after being processed, they will be examined using an X-ray diffraction instrument to determine their structural characteristics and to look at their electrical properties with the four probes method.

3. Results:

When the element pb is partially substituted for the element Bi in the compound $\text{Bi}_{2-x}\text{Pb}_x\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ compound, with $x=0, 0.15, 0.25, \dots$, the structural properties of the compound have been studied. However, the structural properties of the compound $\text{Bi}_{2-x}\text{Pb}_x\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$, which was prepared at an annealing temperature of 850 and under a hydrostatic pressure of 8 tons/cm² are different. When creating the models with the X-ray device and putting the value of x into the compound, the X-ray diffraction investigation of these samples revealed. The regularity of the crystalline structure and the appearance of distinct peaks are noticed as shown in **Figure 1**.

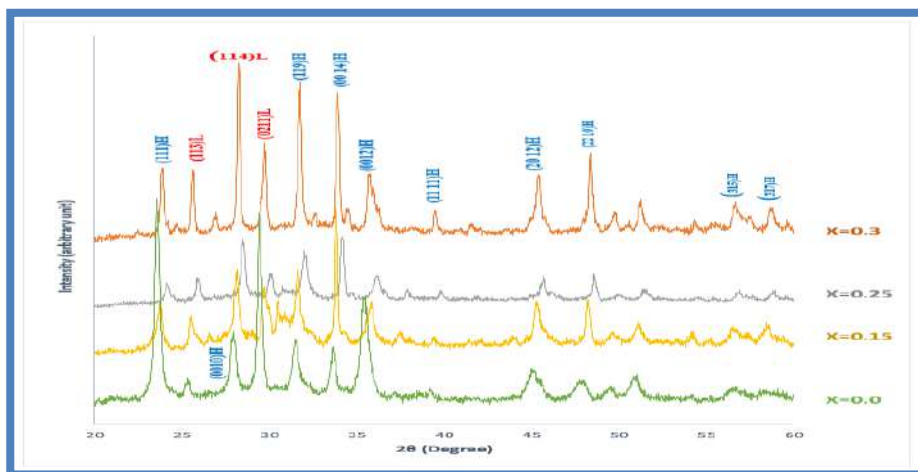


Figure 1. X-ray diffraction of $\text{Bi}_{2-x}\text{Pb}_x\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ compound as function of 2θ , with indicated values of $x=0,0.15,0.25, x=0.3$

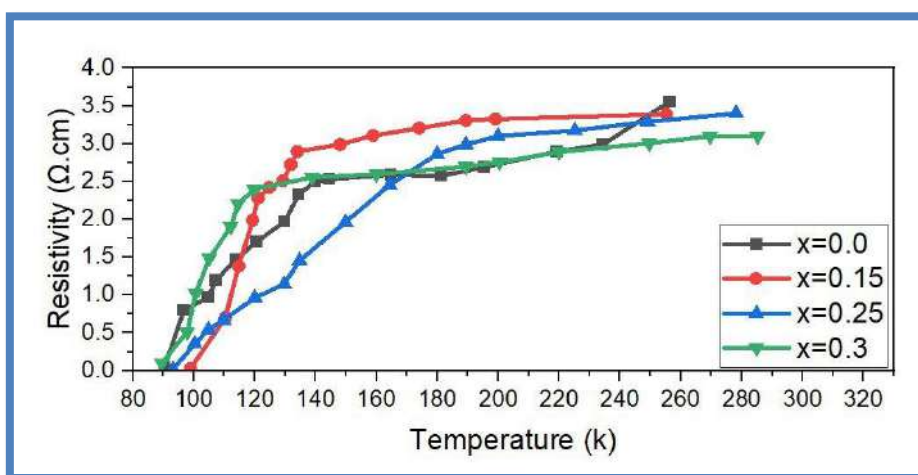


Figure 2: Resistivity of $\text{Bi}_{2-x}\text{Pb}_x\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ compound as function of pb , with indicated values of $x=0,0.15,0.25, x=0.3$

Figure 2 presents the electrical resistance behavior as a function of temperature for the four samples, both before and during the partial replacement. The maximum zero-resistivity (T_c) of 135 K was achieved at $x = 0.25$; however, at other concentrations, T_c increased with x concentration, as shown in Table 1.

Table (1): Values axes a , c , $T_c(\text{onset})$, ΔT and $T_c(\text{offset})$ of $\text{Bi}_{2-x}\text{Pb}_x\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ compound as function of pb , with indicated values of $x=0,0.15,0.25,0.3$

X	a (°Å)	c (°Å)	c/a	Tc (OFF)(K)	Tc (ON) (K)	Tc(K)
0	5.0923	35.017	6.97	95	150	123
0.15	5.339	36.895	6.91	198	160	129
0.25	5.432	37.028	6.81	100	170	135
0.3	4.973	36.962	7.64	99	122	110

4. Discussion

The regularity of the crystalline structure and the appearance of distinct peaks are noticed as shown in **Figure 1**.

Miller coefficients hkl are then discovered, and using a particular BASIC software, the values of the unit cell's dimensions are discovered. $a = b = 5.09 \text{ \AA}$ and, $c = 35.01 \text{ \AA}$ for $x = 0$, where: $a = b = 5.33 \text{ \AA}$, $c = 36.89 \text{ \AA}$ for $x = 0.15$, $a = b = 5.43 \text{ \AA}$ and, $c = 37.02 \text{ \AA}$ for $x = 0.25$, as shown in **Table 1**. The table indicates that elevating the concentration of partially transcendent elements results in a significant alteration of both the network constants and their size. An increase in the value of c resulted in variations in the concentrations $x = 0.0, 0.15, 0.25, 0.3$, attributed to disturbances in the number of CuO chains and CuO₂ levels that govern the various isotropic materials and the T_c of superconducting materials. A decrease in the coefficient c diminishes the intensity of the fermi energy levels, thereby lowering the T_c , causing imbalance and variability, and affecting the unit cell volume in a compound [21]. The change in lattice constants arises from the variation in ionic radii of the substituted elements resulting from an increase in substitution concentration. X-ray diffraction measurements indicated that the crystalline structure remained tetragonal [22], with a noticeable reduction in the c -axis length when the compensation ratio increased to $x = 0.3$, attributed to the relocation of atomic defects or oxygen deficiency. The vacuoles or abnormalities of the positive ions result in the accumulation of stacking faults. The distortion of the crystal structure eventually results from the influence along the (c) axis [23].

As shown in **Figure 2** the results indicate that all samples exhibited metallic behavior, with the T_c rising in accordance with the increasing x concentration. The electrical resistance decreases as the temperature decreases in the region preceding ($T_c(\text{onset})$), as the material transitions from its natural state. The change to the superconducting state occurred in several stages, influenced by several transitions arising from the various phases present in the sample, as well as the presence of certain crystals and impurities. The findings indicated that the T_c varies with increasing x concentration [22]. The maximum zero-resistivity (T_c) of 135 K was achieved at $x = 0.25$; however, at other concentrations, T_c increased with x concentration, as shown in **Table 1**.

This result can be explained by the compound playing the perfect role in the crystal structure [20], and this percentage of compensation caused the T_c to rise as a result of an increase in the high phase 2223 with an increase in the lead concentration in the samples.

Using the four-probe approach, the electrical characteristics of the compound were investigated in order to determine $T_c(\text{onset})$ and $T_c(\text{offset})$ for calculating the electrical resistivity as a function of temperature after the element pb was partially substituted in the Bi element of the $\text{Bi}_{2-x}\text{Pb}_x\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ compound.

5. Conclusions

The effect of partial bismuth replacement with lead on the superconducting characteristics of $\text{Bi}_{2-x}\text{Pb}_x\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ is examined in this publication with $x = 0, 0.15, 0.25$ and 0.3 . The solid-state reaction approach was used to prepare the samples. The four-probe method was used to evaluate the electrical conductivity in order to calculate the T_c . The T_c was observed to rise as the quantities of silver in all produced samples increased. The optimal compensation ratio for x was also discovered to be 0.25 , which results in the highest T_c . X-ray diffraction was used to examine the samples' ultrastructural characteristics. All of the prepared samples had a tetragonal structure, with a distinct change in the lattice constants, according to an X-ray diffraction investigation. The ideal role that the compound played in the crystal structure can be utilized to explain this result. This percentage of compensation led to an increase in the T_c and an increase in the high phase 2223 with an increase in the lead concentration in the samples.

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Determination of Heavy Metals in Selected Types of Local and Imported Tea from Iraqi Markets

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Keywords: Heavy Metals; Toxicity; Black Tea; Green Tea; EDX; Atomic Absorption.

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Abstract:

The presence of heavy metals in plants, including tea plants, is influenced by factors such as the plant's origin, geographic location, soil geochemistry, and environmental contaminants in soil, water, and air. Elevated concentrations of heavy metals pose serious health risks to humans, including the potential for various cancers and damage to vital organs such as the liver, kidneys, and brain. In this study, seven tea samples were collected from the Iraqi market to examine their safety for consumption, indicated as T1-T7. The selection of the samples is based on a questionnaire answered by 140 individuals; T1-T6 samples are black tea, and T7 is green tea. Although the primary objective of this research is to determine the heavy metal content and toxicity of all the tea samples, the secondary one is to assess whether the drying process affects the levels of heavy metals by comparing green and black tea. The analysis is conducted on the samples via the Energy-Dispersive X-ray (EDX) and atomic absorption spectroscopy (AAS). The EDX results showed the presence of S, K, Al, and P in the T1 sample, while K, Al, and Mg

were identified in the T2 sample, S, K, Al, and P in T3, only K in T4, K and Fe in T5, K, P, and S in T6, and Only K in T7. The results obtained from the AAS showed that all samples have normal concentrations of Fe and Zn but not for Ni (in samples T1, T2, and T3) and manganese (in samples T1, T2, T4, T5, T6, and T7). Those detected concentrations are higher than the permissible levels, and their levels are within the toxic limits according to the standard limits set by the World Health Organization (WHO) and the Food and Agriculture Organization (FAO). However, the other heavy metals (Cd, Cu, Pb, Cr, and Co) were either not detected or were below the detection limits of AAS, indicating that these samples are free from the mentioned toxic heavy metals.

Keywords: Heavy Metals; Toxicity; Black Tea; Green Tea; EDX; Atomic Absorption

(Immediately after the abstract, provide 5-7 keywords and arrange them alphabetically, using American spelling and avoiding general and plural terms and multiple concepts (avoid, for example, 'and', 'of'). Be sparing with abbreviations: only abbreviations firmly established in the field may be eligible. These keywords will be used for indexing purposes).

تحديد العناصر الثقيلة في أنواع مختارة من الشاي المحلي والمستورد من الأسواق العراقية

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الخلاصة:

يتأثر وجود المعادن الثقيلة في النباتات، بما في ذلك نباتات الشاي، بعوامل عدة مثل أصل النبات، والموقع الجغرافي، والكيمياء الجيولوجية للتربة، والملوثات البيئية في التربة والمياه والهواء. وتشكل التراكيز المرتفعة من هذه المعادن مخاطر صحية جسيمة على الإنسان، بما في ذلك احتمال الإصابة بأنواع مختلفة من السرطان وتلف الأعضاء الحيوية مثل الكبد والكلى والدماغ. في هذه الدراسة، تم جمع سبع عينات من الشاي من السوق العراقية لفحص مدى سلامتها للاستهلاك، أُشير إليها بالرموز T1 إلى T7، وتم اختيار العينات بناءً على استبيان شمل ١٤٠ فردًا، حيث كانت العينات T1-T6 من الشاي الأسود، بينما كانت العينة T7 من الشاي الأخضر. يهدف البحث إلى تحديد محتوى المعادن الثقيلة وسمية جميع العينات،

مع تقييم تأثير عملية التجفيف على مستويات المعادن الثقيلة من خلال مقارنة الشاي الأسود بالشاي الأخضر. تم تحليل العينات باستخدام تقنية الأشعة السينية للطاقة المشتتة (EDX) وتقنية مطيافية الامتصاص الذري (AAS)، حيث أظهرت نتائج تحليل EDX وجود عناصر مثل الكبريت والبوتاسيوم والألمنيوم والفوسفور في العينة T1، وعناصر البوتاسيوم والألمنيوم والمغنيسيوم في العينة T2، وعناصر الكبريت والبوتاسيوم والألمنيوم والفوسفور في العينة T3، والبوتاسيوم فقط في العينة T4، وعناصر البوتاسيوم والحديد في العينة T5، وعناصر البوتاسيوم والفوسفور والكبريت في العينة T6، والبوتاسيوم فقط في العينة T7. أما نتائج تحليل AAS، فقد أظهرت أن جميع العينات تحتوي على تراكيز طبيعية من الحديد والزنك، بينما تجاوزت تراكيز النيكل في العينات T1 وT2 وT3، وتراكيز المنغنيز في العينات T1 وT2 وT4 وT5 وT6 وT7 الحدود المسموح بها وفقاً للمعايير التي وضعتها منظمة الصحة العالمية (WHO) ومنظمة الأغذية والزراعة (FAO)، مما يشير إلى وجود مستويات سامّة من هذه المعادن. ومع ذلك، لم تُكشف المعادن الثقيلة الأخرى (الكاديوم، النحاس، الرصاص، الكروم، الكوبالت) أو كانت تراكيزها أقل من حدود الكشف باستخدام تقنية AAS، مما يدل على أن هذه العينات خالية من المعادن الثقيلة السامة المذكورة.

الكلمات المفتاحية: المعادن الثقيلة، السمية، الشاي الأسود، الشاي الأخضر، EDX، الامتصاص الذري.

1. Introduction:

Camellia Sinensis, globally renowned as the most popular botanical source of tea, is cherished for both its flavor and reputed therapeutic benefits [1]. Tea drinking has deep historical roots, dating back approximately five millennia, as evidenced by ancient tales from China and India [2]. In Iraq, tea holds significant cultural significance, being enthusiastically embraced and routinely savored across various occasions and gatherings. Particularly, Iraqi Kurds exhibit a remarkable fondness for tea; the annual consumption rate is around 1.5 kg per person, positioning the Kurdistan region among the world's top four areas, alongside Turkey, Ireland, and the United Kingdom [3, 4]. Traditionally, tea has been imbibed for its purported abilities to enhance circulation, detoxify the body, and bolster resistance against illnesses [5].

Tea originates from the leaves of the Camellia Sinensis shrub [6], which thrive in regions characterized by high humidity, moderate temperatures, acidic soils, and spanning from sea level to high-altitude mountains [7,8]. While both black and green teas are derived from the same plant, their distinct appearance, flavor, and aroma are remarkably different. This is attributed to the variations in the dryness process during tea production. This process imbues teas with floral, spicy, or fruity notes, resulting in discernible differences between black and green teas based on their processing methods and oxidation levels [9]. Nonetheless, it's important to note that tea leaves are prone to absorbing heavy metals (HMs) throughout their lifecycle, from cultivation to packaging and transportation, potentially leading to increased metal exposure in consumers [10]. Among the common heavy metals that tea leaves may

accumulate are Cadmium (Cd), Copper (Cu), Lead (Pb), Chromium (Cr), Nickel (Ni), Iron (Fe), Manganese (Mn), mercury (Hg), Zinc (Zn), and Cobalt (Co). [11].

Heavy metals are natural elements characterized by their relatively high atomic mass and density; they play integral roles in the food chain, various human activities, and consumption patterns. Heavy metals are characterized by their relatively high atomic mass and density, occurring naturally in varying concentrations across the Earth's crust. Typically defined by a density of at least 5 g/cm³, HMs can also be distinguished by an atomic mass greater than 23 or an atomic number surpassing 20 [12]. In biological systems, heavy metals are classified as essential or nonessential. In contrast, essential heavy metals, such as Mn, Fe, Cu, and Zn, are vital for living organisms and are required in trace amounts. Contrarily, nonessential heavy metals, like Cd, Pb, and Hg, are toxic and lack biological significance [13-15]. The accumulation of HMs in tea plants is influenced by various factors, including the plant's origin, geographic location, soil geochemistry, and environmental contaminants present in soil, water, and air [16, 17]. Soil contamination sources, such as industrial activities, pesticide and fertilizer usage, and untreated wastewater, also contribute to elevated levels of HMs. Consequently, tea plants absorb these metals from the soil, potentially leading to high concentrations in edible parts, such as leaves [18-20]. While naturally occurring in soil, mining and smelting activities can exacerbate contamination levels. Here, tea plants, with their ability to uptake HMs from the soil, may transfer certain amounts to the leaves that are ultimately used in tea production [21-23].

2. Material and methods:

2-1 Sample Selection:

The sample selection method was based on the availability of products in the Iraqi market. However, three critical factors were considered while selecting the samples. Firstly, the product's popularity was considered, which ensured that only commonly used products were included in the study. Thus, a questionnaire was created and distributed online to individuals from varying economic backgrounds, and over 140 responses were obtained. This approach helped to establish a clear understanding of the popularity of the products within the target population and aided in the selection of products for the study. Figure 1 shows the results of the public questionnaire.

The second factor was the geographical location, which played a crucial role in ensuring a diverse range of products was collected. Finally, the price of the product was also considered to ensure that a range of products, from low to high prices, were included in the study. The combination of these three factors helped to provide a diverse range of samples that were

representative of the Iraqi market. Furthermore, one of the samples was green tea instead of black tea for broader exploration.

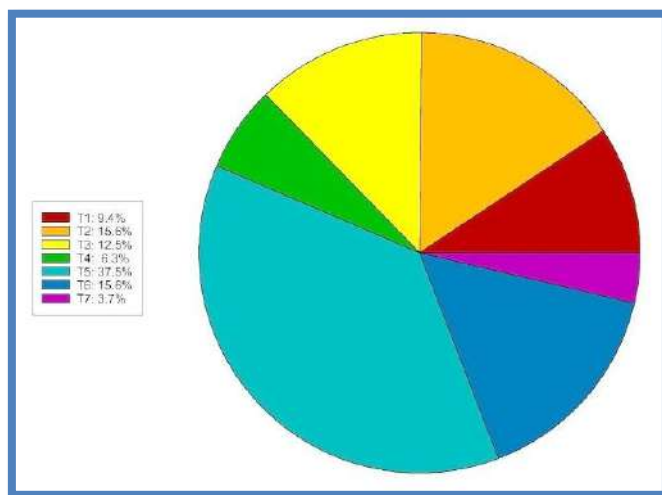


Figure 1: The public questionnaire results of selected samples.

2-2 Reagents and Chemicals

2-2-1 Reagents Used in Acid Digestion: Sample preparation involved analytical-grade nitric acid (65%, Sigma Aldrich) and hydrochloric acid (37%, Sigma Aldrich) [24, 25].

2-2-2 Reagents Used in Atomic Absorption: All the solutions were prepared using deionized water [26, 27]. Standard solutions for calibration of Mg, Zn, K, Cu, Cr, Ni, Co, Cd, Pb, and Fe were prepared from 1000 mg/L (ppm) Standard Stock Solution of GFS Fishers' AAS Reference Standard. Dilution correction was applied for samples diluted or concentrated during analysis.

2-3 Samples Drying

The tea samples, each weighing 1 g, were individually placed in watch glasses. Subsequently, they were dried in an electrical oven at 100°C overnight. Hence, they were ensured to be completely dry and ready for use in acid digestion and for Energy-Dispersive X-ray (EDX) spectroscopy analysis.

2-4 EDX Analysis

An EDX of Bruker model XFlash6110 was used to analyze the chemical characterization/elemental analysis of materials.

2-5 Acid Digestion Method

A 0.5 g of the selected dry sample was placed separately into 200 ml beakers, then 15 ml of 65% HNO₃ and 10 mL of 37% HCl were added [24, 25, 28]. The contents were mixed by stirring thoroughly and heated on a hot plate. The heating continued until the dissolution of the content, and the digested sample was then filtered using Whatman filter paper No.41. The

filtration solution was diluted with deionized water to 100 ml. The resulting solution was used for the spectrophotometric determination of various metals using atomic absorption spectroscopy (AAS). In accordance with chemical laboratory safety rules, this procedure was conducted in a fume hood.

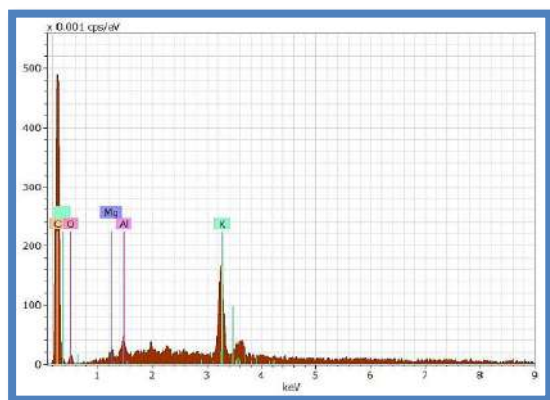
2-6 Atomic absorption Spectroscopy (AAS) Analysis

An Agilent AAS of the FS240 model was used to determine the content of heavy metals and their concentrations in the previously acid-digested prepared samples. To determine the concentration of heavy metals (HMs) in the samples, the results were multiplied by a calculation factor obtained by dividing 100 mL (the volume used for sample dilution following digestion) by 0.5 g (the quantity of tea sample used for the analysis).

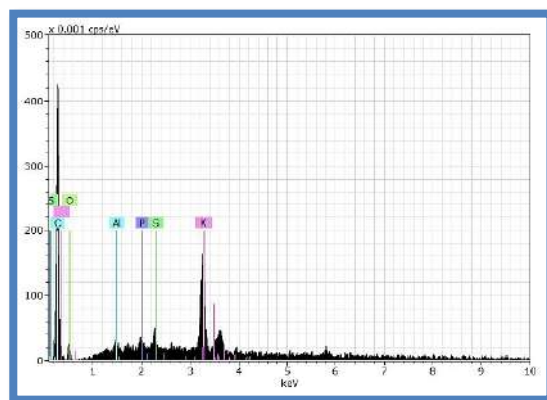
3. Results and Discussion:

3-1 Energy-Dispersive X-ray (EDX) Analysis

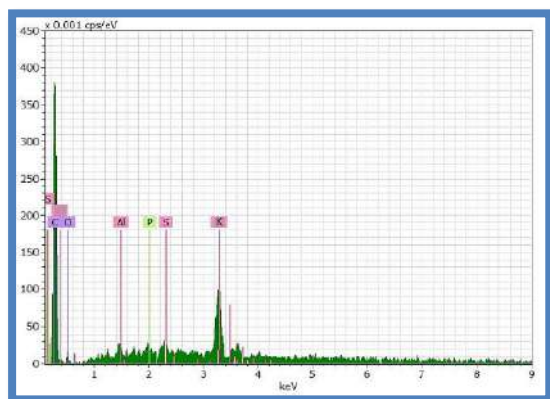
The six black tea samples (T1-T6) and the green tea sample (T7) selected from Iraqi markets were tested via EDX spectroscopy for elemental determination. The normal scan results obtained for this test were illustrated in **Figures 2**, a-g, respectively. The elements shown in the figures were in concentrations higher than 0.003 cps/ev.



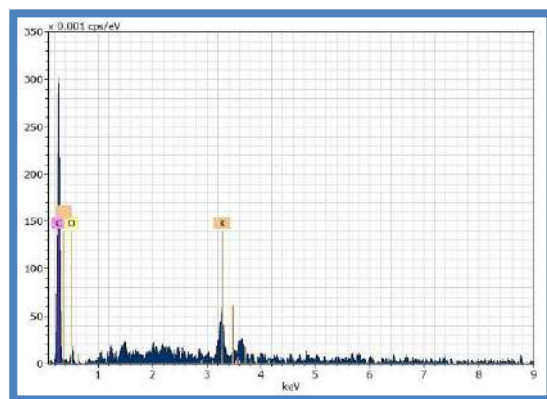
(a)



(b)



(c)



(d)

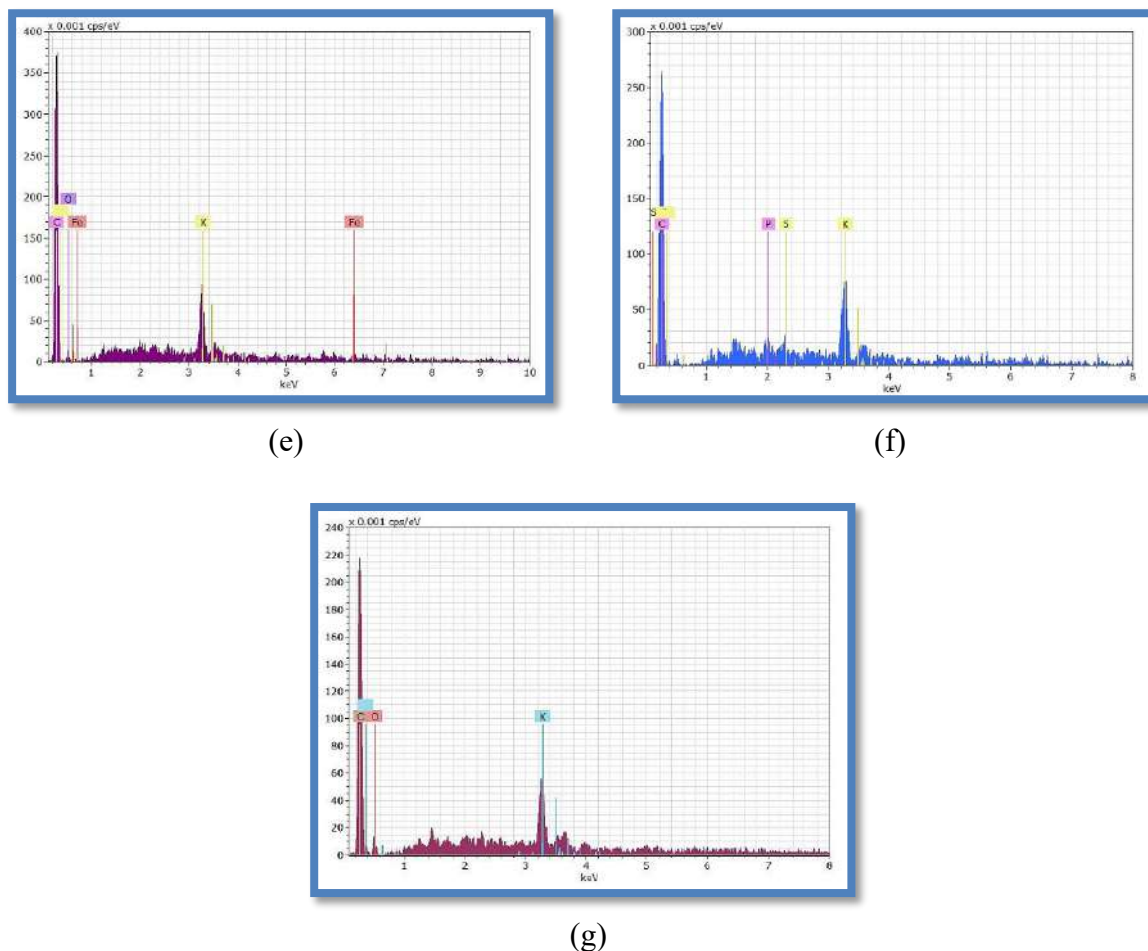
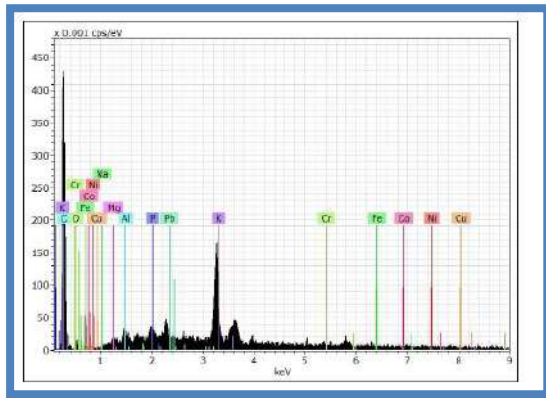


Figure 2: EDX Analysis for Samples (a) T1, (b) T2, (c) T3, (d) T4, (e) T5, (f) T6, and (g) T7.

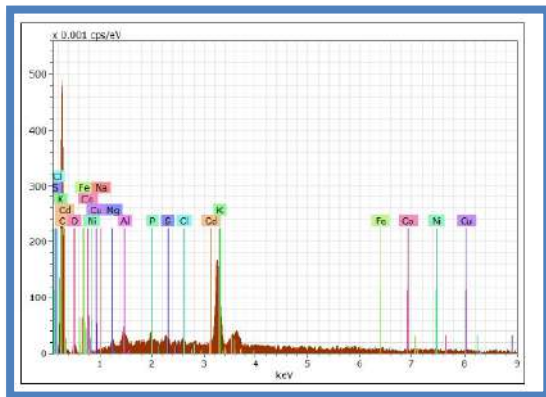
A normal scan of EDX analysis shows the presence of S, K, Al, and P in T1, while K, Al, and Mg were found in T2. Furthermore, S, K, Al, and P were found in T3, but only K was detected in T4. K and Fe were detected in T5, K, and P, and only S was identified in T6, and only K was found in T7. All of the elements that were diagnosed above and detected are considered essential elements, which are necessary for living organisms and may be required in the body in low concentrations. However, the other elements were detected to be within the acceptable range (concentrations lower than 0.003 cps/ev) as shown in [Figure 3](#) a and b for T1, c and d for T2, e and f for T3, g and h for T4, i and j for T5, k and l for T6, and m and n for T7. To determine the exact concentrations of elements identified by the EDX device, along with those indicated by other research studies but possibly present in a concentration too low to be detected by the EDX due to its sensitivity limits, a digestion process was conducted on the selected samples. Acid digestion using a mixture of 65% HNO₃ and 37% HCl was performed to release HMs from the organic components in tea samples. The resulting solutions were then tested using an atomic absorption instrument to detect and determine the concentrations of the metals.



(a): Accurate EDX Analysis for T1.



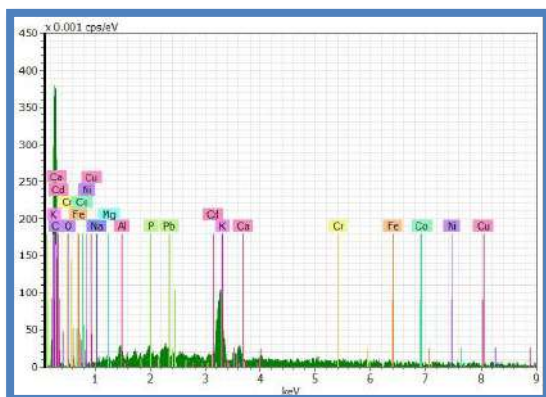
(b): Accurate EDX Analysis for T1.



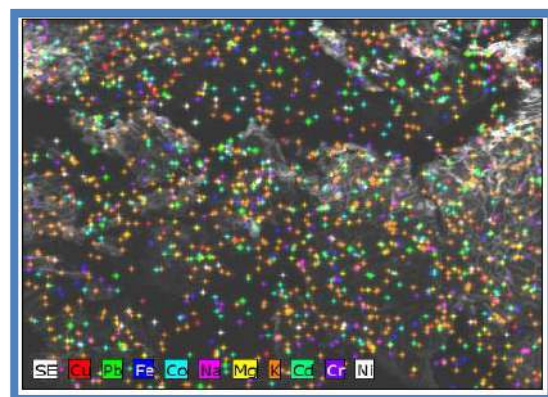
(c): Accurate EDX Analysis for T2.



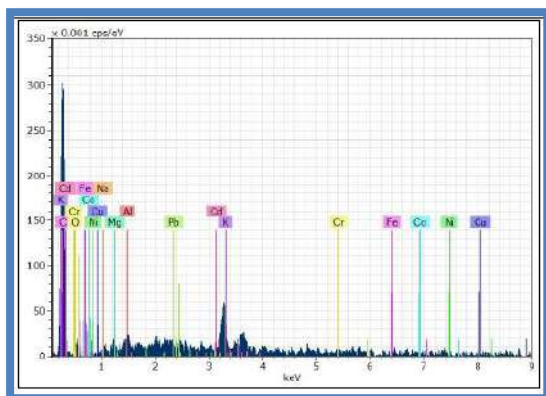
(d): Elements distribution in T2.



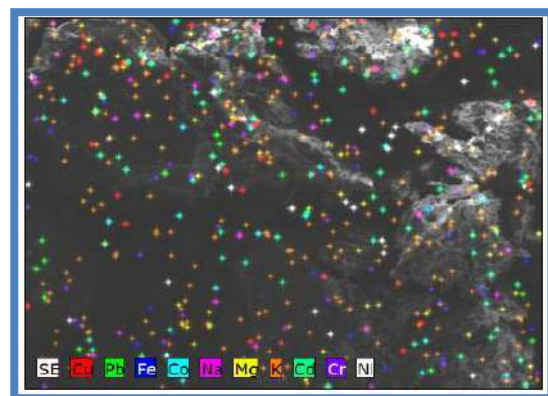
(e): Accurate EDX Analysis for T3.



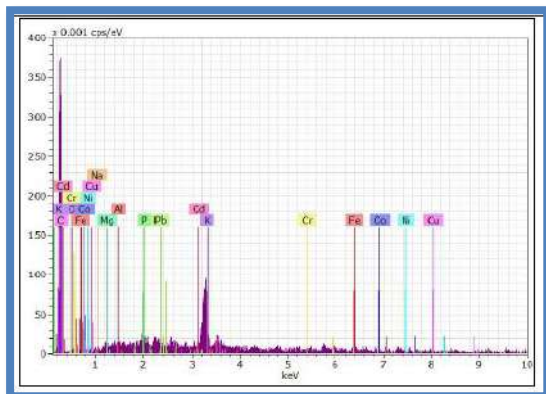
(f): Accurate EDX Analysis for T3.



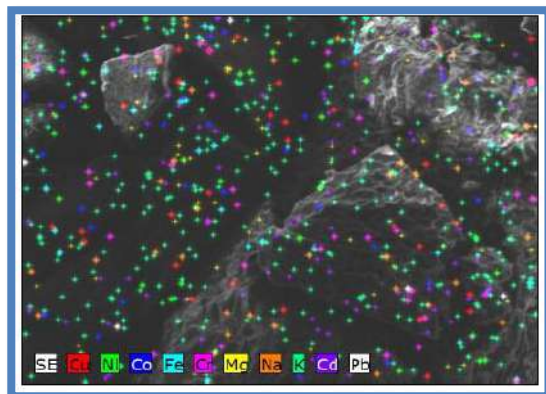
(g): Accurate EDX Analysis for T4.



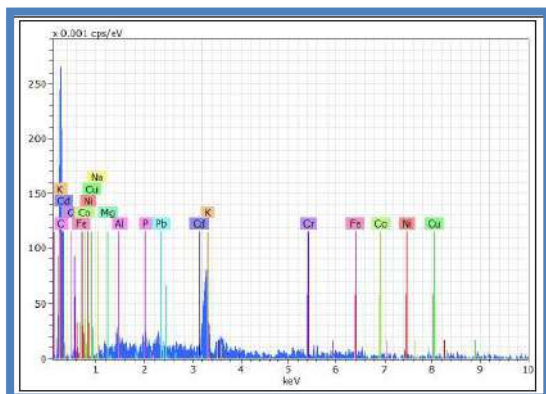
(h): Accurate EDX Analysis for T4.



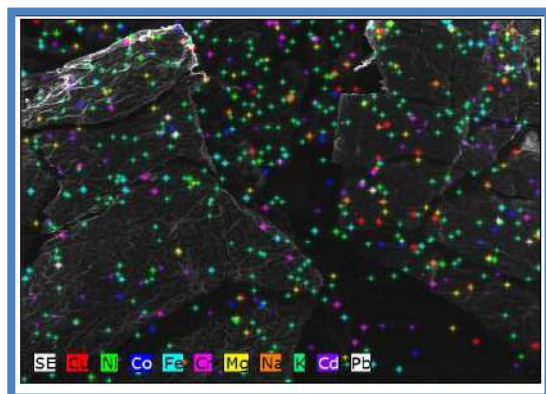
(i): Accurate EDX Analysis for T5.



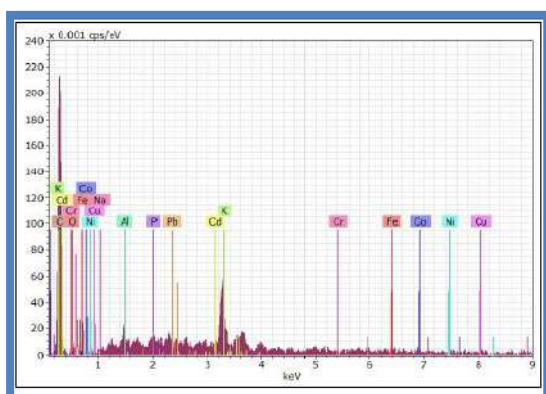
(j): Accurate EDX Analysis for T5.



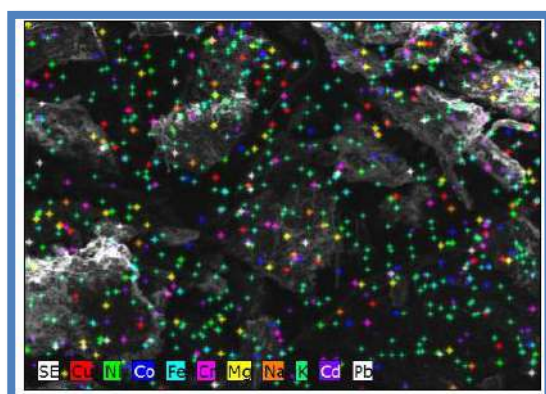
(k): Accurate EDX Analysis for T6.



(l): Accurate EDX Analysis for T6.



(m): Accurate EDX Analysis for T7.



(n): Accurate EDX Analysis for T7.

Figure 3: Accurate EDX Analysis for Samples a and b for T1, c and d for T2, e and f for T3, g and h for T4, i and j for T5, k and l for T6, and m and n for T7.

3-2 AAS Analysis

In order to establish a calibration curve for a specific metal within the detection limits of a device, four standard solutions with low concentrations (in ppm) of nine HMs and one essential element were prepared. Subsequently, solutions derived from the digestion of seven tea samples were compared with these standards. **Table 1** shows the final results for the selected elements in tea samples, while the ratios of metal are illustrated in **Table 2**.

Table 1: AAS Analysis results in ppm

Sample Element	T1	T2	T3	T4	T5	T6	T7
K	14000	14630	10178	6602	14000	12460	8260
Cd	ND	ND	ND	ND	ND	ND	ND
Cu	ND	ND	ND	ND	ND	ND	ND
Pb	ND	ND	ND	ND	ND	ND	ND
Cr	ND	ND	ND	ND	ND	ND	ND
Ni	46	46	66	ND	ND	ND	ND
Fe	500	500	376	376	376	ND	300
Mn	1866	666	332	932	566	466	1000
Zn	54	10	22	66	4	10	ND
Co	ND	ND	ND	ND	ND	ND	ND

ND= Not detected

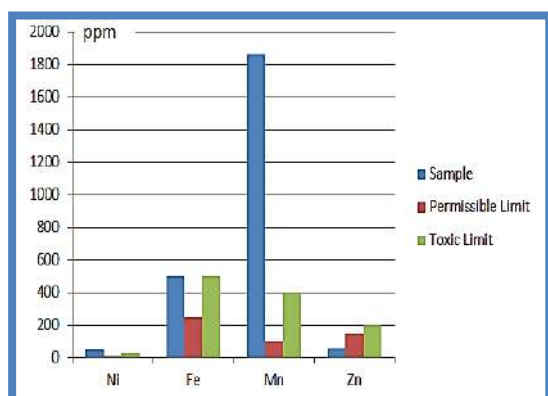
Table 2: The percentages (%) of HMs and potassium in tea samples

Sample Element	T1	T2	T3	T4	T5	T6	T7
K	1.4	1.46	1.01	0.66	1.4	1.24	0.82
Ni	0.0046	0.0046	0.0066	ND	ND	ND	ND
Fe	0.05	0.05	0.037	0.037	0.037	ND	0.03
Mn	0.18	0.06	0.03	0.09	0.06	0.05	0.1
Zn	0.005	0.001	0.002	0.007	0.0004	0.001	ND

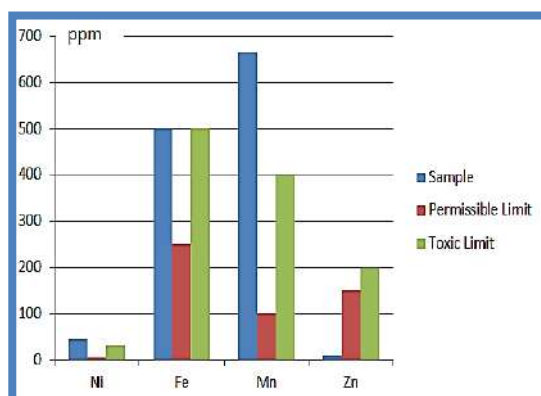
According to the standard limits given by the World Health Organization (WHO) and Food and Agriculture Organization (FAO) (as presented in **Table 3**) and the results obtained from the AAS, all samples (6 black tea and the seventh green tea) had normal concentrations of Iron (it exceeded the permissible limit but did not exceed the toxic level). Also, Zinc and nickel (in samples T1, T2, and T3) and manganese (in samples T1, T2, T4, T5, T6, and T7) were higher than the permissible levels, and their concentrations were within the limits of toxic concentrations. The other HMs (Cd, Cu, Pb, Cr, and Co) were either not detected or were less than the detection limits of these metals in atomic absorption, which makes these samples free from highly toxic HMs. Metal concentrations for samples (T1-T7) compared with permissible and toxic concentrations are illustrated in **Figure 4** (a-g).

Table 3: Normal and Toxic concentrations of heavy metals given by WHO and FAO [29, 30].

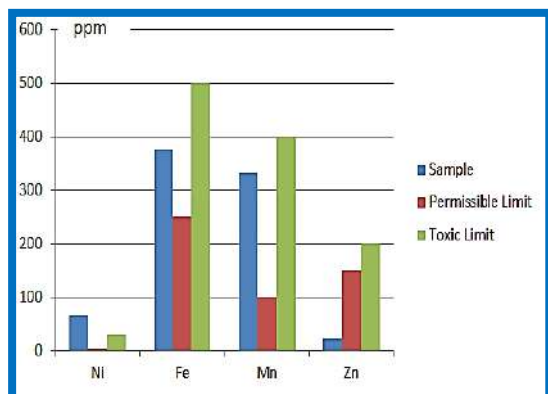
Element	Normal concentrations (mg kg ⁻¹)	Toxic concentration (mg kg ⁻¹)
Cu	3–15	20
Ni	0.1–5	30
Pb	1–5	20
Hg	<0.1–0.5	5
Cr	<0.1–1	2
Mn	15–100	400
Zn	15–150	200
Mo	0.1–0.5	10–50
Co	0.05–0.5	30–40
Fe	50–250	(>500)
As	10–60	<2
Sb	<2–29	5–10 g



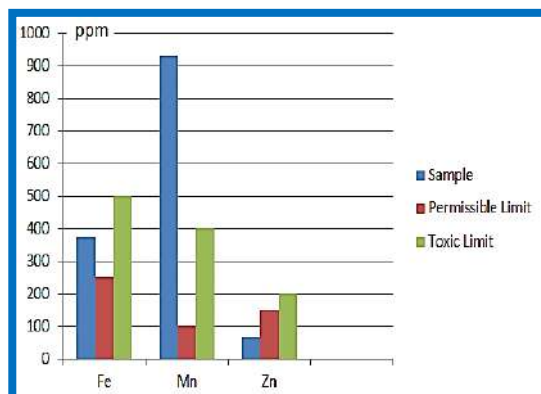
(a): Metal concentration of the sample(T1)



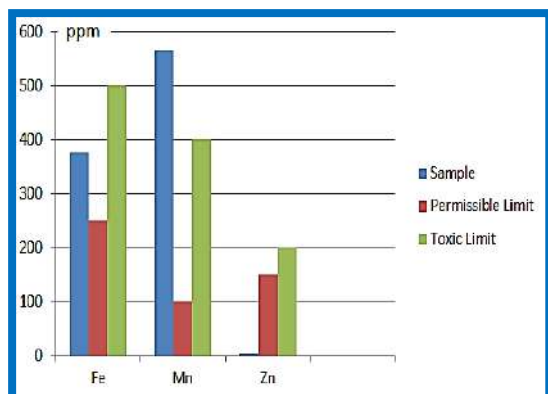
(b): Metal concentration of the sample(T2)



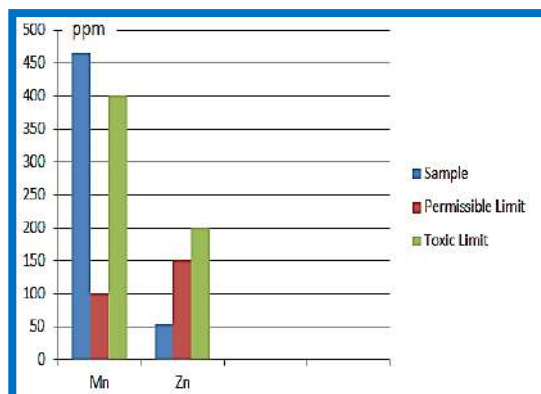
(c): Metal concentration of the sample(T3)



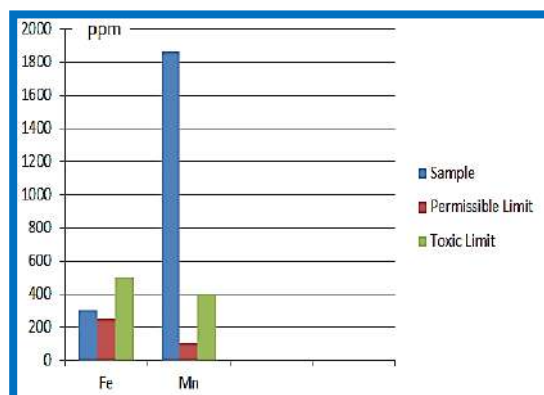
(d): Metal concentration of the sample(T4)



(e): Metal concentration of the sample(T5)



(f): Metal concentration of the sample(T6)



(g): Metal concentration of the sample(T7)

Figure 4: Metal concentrations for samples (a) T1, (b) T2, (c) T3, (d) T4, (e) T5, (f) T6, and (g) T7 compared with permissible and toxic concentrations.

Potassium is the third most important plant nutrient, and it's the second most important nutrient after nitrogen for tea. Tea requires a comparatively higher quantity of K for better production and quality of the product. Here, K plays a vital role in enzyme activation, water relation, translocation, energy relations, and translocation of assimilated photosynthesis and protein and starch synthesis. Potassium's concentrations are clearly higher than the other elements, but potassium is considered an electrolyte mineral that helps regulate heartbeat and balance the body's fluid levels. It can help offset the adverse effects of consuming too much sodium [31].

According to research published by the University of Maryland Medical Center, a cup of green tea supplies lower potassium concentrations than black tea [32]. In our work, the results proved that the potassium concentration of sample 7 (the green tea) had a lower potassium level than 83.3% of the black tea samples. From the EDX Analysis, sulfur, phosphorus, aluminum, and magnesium were detected in too-low concentrations that were placed within the safe range. Additionally, it is essential to mention that the selected samples did not contain HMs like Cd, Pb, and Hg, which are toxic and are regarded as biologically non-essential in remarkable quantities.

4. Conclusions

Tea is considered one of the foodstuffs commonly used globally and locally in Iraq, and the high consumption rates make it at the forefront of foodstuffs that require knowledge of its suitability for human consumption. Some consumer concerns are generated about the potential risks of consuming tea, especially with poor control over imported food. To dispel doubts or prove them, data were obtained in this work to determine the validity of the food product concerning its containment of HMs within the permissible limits according to international health and safety standards. Here, seven samples of tea types commonly used in the Iraqi market were selected. Processes of preparing the samples for specialized examinations of HMs (EDX and AAS) were conducted according to the scientific protocols related to them. Results of these tests show that all the selected samples had high concentrations of nickel (in samples T1, T2, and T3) and manganese (in samples T1, T2, T4, T5, T6, and T7). The other HMs (Cd, Cu, Pb, Cr, and Co) were not detected or found in levels less than the detection limits of the used AAS, which makes these samples free from highly toxic HMs. Researchers recommend studying the chemicals in tea and discussing their toxicity. We also recommend the necessity of examining imported materials to ensure that they are free of toxic materials or materials that affect human health.

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5. References

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